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EFFECT OF HOM FREQUENCY SHIFT ON BUNCH TRAIN STABILITY

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Abstract

Future operation of the Large Hadron Collider (LHC) at CERN foresees an increase of the nominal luminosity by a factor of ten. The High-Luminosity LHC (HL-LHC) project necessitates good quality beam with increased intensity delivered by the injector chain. An important quality of the beam is its stability. In the LHC injector, the Super Proton Synchrotron (SPS), the longitudinal Coupled-Bunch Instability (CBI) during the acceleration ramp is a limiting factor in the production of high intensity beams. The main source of CBI is the 630 MHz Higher-Order Mode (HOM) in the 200 MHz travelling wave RF cavities. This HOM is already heavily damped by a series of longitudinal RF couplers. To achieve greater levels of damping, additional methods are under investigation using electromagnetic simulations. It has been shown in particle simulations that shifting the resonant frequency of the HOM can significantly improve the beam stability by up to 50%. The dependence of the instability threshold with the resonant frequency of the 630 MHz HOM is studied for an SPS bunch train of 72 bunches. The results of the macro-particle simulations are substantiated with an analytical model.

INTRODUCTION

Two independent travelling wave RF systems operate in the SPS for the production of the LHC proton beams. The acceleration system at 200 MHz [1] is supported by a second RF system at 800 MHz, operating as a Landau system in bunch-shortening mode [2]. Moreover, for nominal LHC intensity, the bunch emittance is blown-up in a controlled way during acceleration to ensure beam stability. However, this will not be sufficient to guarantee the stability of the HL-LHC beam. The acceleration system must therefore undergo significant upgrades. The available power per cavity will be raised and the number of cavities increased [3]. The new cavities are shorter than the existing ones to minimise power requirements and impedance. The updated acceleration system consists of two four-section and four three-section travelling wave RF systems [4]. Each section is made of eleven cells where RF probes and RF couplers are installed. Due to beam-loading in the cavities, a maximum voltage of 10 MV is only achievable at flat top (450 GeV) for an intensity of 2.4×10^{11} particles per bunch (ppb). In addition, a redesign of the low-level control of the 200 MHz RF system reduces further the beam-loading. Feedback and feedforward systems are part of the 200 MHz low-level RF and are expected to reduce the impedance seen by the beam up to a factor of 26 dB in the accelerating band after upgrade.

In the cavities an HOM in the 630 MHz passband is critical for beam stability. The mode is already heavily damped by a series of RF couplers and further damping is difficult to achieve without modifying the layout of the cavity. The natural spread of the HOM from cavity to cavity has been measured to be around 100 kHz [5]. The total shunt impedance seen by the beam is not sufficiently reduced by this spread and detuning the mode differently for each cavity, mechanically or by using new RF couplers, has been proposed as a solution for improved stability. Particle simulations show that a frequency spread of the HOM from cavity to cavity by a few MHz could already significantly improve the situation. Moreover, a full detuning of the mode by 10 MHz gives a major increase up to 50% of the intensity threshold. However, the mode in the 630 MHz passband appears to be rigid. Most of the stored energy is in the cavity volume [6] and the frequency cannot be shifted by 10 MHz with RF couplers. To understand the possible gain of a smaller shift, the mechanism behind the improvement of beam stability is studied in what follows.

The total HOM of the four three-section cavities and two four-section cavities without further damping can be modelled by a resonator with a shunt impedance $R_{sh} = 570$ kΩ and a quality factor $Q = 200$, with impedance

$$Z(f) = \frac{R_{sh}}{1 + iQ(\frac{f}{f_r} - \frac{f_r}{f})} \tag{1}$$

and

$$W(\tau) = \begin{cases} \alpha R_{sh} & \text{if } \tau = 0, \\ 2\alpha R_{sh} e^{-\alpha\tau}\left(\cos(\bar{\omega}\tau) - \frac{\alpha}{\bar{\omega}}\sin(\bar{\omega}\tau)\right) & \text{if } \tau > 0, \end{cases} \tag{2}$$

is the Green's function of the electromagnetic field of the resonator with parameters

$$\alpha = \frac{\pi f_r}{Q} \text{ and } \bar{\omega} = 2\pi f_r \sqrt{1 - \frac{1}{4Q^2}}. \tag{3}$$

$W(\tau)$ is also called the wake function and its Fourier transform is the impedance. The e-folding time of the wake function generated by the HOM is

$$\frac{1}{\alpha} = \frac{Q}{\pi f_r} \approx 100 \text{ ns.} \tag{4}$$

In the LHC beam, bunches are separated by 25 ns, therefore, several bunches are coupled through their wakefield which leads to possible CBI depending on beam intensity.

FREQUENCY DEPENDENCE OF INSTABILITY THRESHOLD

The CBI threshold is the lowest at SPS flat top. This statement is supported by measurements and analytical esti-

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mations [7]. Simulations and analysis below are then made at an energy of 450 GeV. The full SPS impedance model is used [8], containing various resonating elements, broad- and narrow-band, between 50 MHz and 4 GHz. The major contributors to the impedance model are the 200 MHz TW cavities. Both the accelerating band and HOMs bands contribute significantly. The fundamental passband is reduced by the feedback system. Two other significant contributions to the longitudinal impedance arise from the kicker magnets, with broad-band impedance, and the vacuum flanges acting mainly at high frequency (>1 GHz). Many smaller contributions are taken into account, from beam instrumentation devices to the resistive wall impedance. The impedance of these devices has been simulated and/or measured over many years. The complexity of the model makes analytical estimations difficult and particle tracking codes like BLonD [9], developed in the RF group at CERN, is used to study the impact on beam stability of the different components of the longitudinal SPS impedance model. The intensity threshold is determined performing scans of both intensity and longitudinal bunch emittance. Particle positions in phase space are tracked during a 2 seconds cycle assuming 10 MV at 200 MHz and 1 MV at 800 MHz for each intensity. The bunch distributions are generated in agreement with beam measurements in the SPS. Bunch profiles are computed by projection of the bunch phase space on the time axis and the quadrupole oscillations are extracted. The maximum amplitude of the bunch length oscillations (normalised by the average) is used to separate stable beam from the unstable one. The 630 MHz HOM excites dipole and quadrupole modes at the SPS flat top and both appear in simulation. Usually, bunch length oscillations are used as a criteria to define the stability threshold in simulations of bunch trains with the full impedance model.

Figure 1 shows the stability threshold for five different values of the HOM resonant frequency between 620 MHz and 640 MHz. In operation a spread in bunch length of ± 8% is measured from bunch-to-bunch along the batch [10] which enforces some necessary margins on the intensity threshold. From simulations with the original HOM at 630 MHz it is observed that beam stability cannot be ensured at HL-LHC intensity. The stability threshold goes up if the HOM is damped by a factor of three keeping R_{sh}/Q constant. For an intensity lower than HL-LHC intensity, the upgraded RF power will allow to increase the voltage above 10 MV. The damping of the HOM by a factor of three is almost sufficient to guarantee beam stability. However, since the damping is difficult to achieve, other solutions were investigated. Shifting the original 630 MHz frequency by a few MHz improves stability as well. The HOM currently sits in an asymmetric region of reduced beam stability. A frequency shift in the positive direction increases stability further than a shift in the negative direction. For a resonant frequency of 640 MHz, simulations show a remarkable improvement of the stability threshold. This frequency corresponds to one of the beam spectrum lines related to bunch spacing (40 MHz). It is well known for a ring filled with equally spaced bunches that a narrow-band impedance with a frequency sitting exactly on a beam spectral line cannot drive instability [11] but the overlap of the beam spectrum and the impedance increases power loss and heating. However, the 630 MHz HOM, already heavily damped to $Q = 200$, is not particularly narrow-band and a train of 72 bunches occupies only height percent of the machine. The first and last bunch of the beam are not coupled and beam spectrum lines are broader than in the full ring case. Nevertheless, simulations exhibit a similar improvement of the stability threshold. Contrary to the ideal case of a full ring the stability must be studied by solving the equations of motion bunch after bunch [12]. Due to the complexity of the machine impedance we choose to isolate the impedance of the 630 MHz and to simplify the equations by considering a single 200 MHz RF system. However, simulations in single and double RF systems give comparable results.

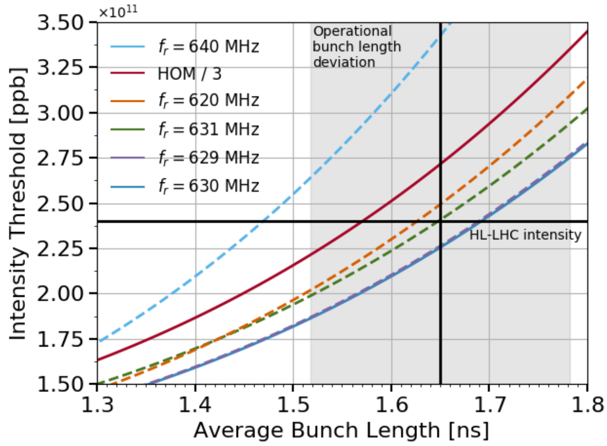

Figure 1: Stability threshold for 72 bunches at SPS flat top in a double RF system with 10 MV for the 200 MHz RF system and 1 MV for 800 MHz RF system in bunch-shortening mode. The full SPS impedance model is used and the resonant frequency of the HOM in the 630 MHz passband is shifted by different values. The horizontal line indicates the HL-LHC beam intensity and the vertical one the nominal bunch length. This bunch length has a spead in operation of ± 8%, indicated by the shaded area.

STABILITY OF TRAINS WITH POINT-LIKE BUNCHES

A point-like bunch model is already able to explain certain observations made in simulation for the LHC beam with a realistic bunch distribution. In such a model each one of the M bunches is represented by a single rigid particle carrying the total bunch intensity. The bunch i oscillates around its synchronous phase ϕ_i with a relative position in time $\tau_i = \frac{\phi_i}{\omega_{rf}}$ where ω_{rf} is the angular frequency of the RF system (Fig. 2). Initially centred in the RF bucket, bunches are separated by a distance τ_{bb}. The equations of motion are derived for small amplitude synchrotron oscillations without

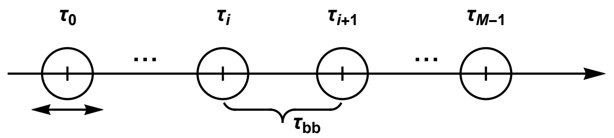

Figure 2: Point-like bunch model. Each bunch is represented by one rigid particle carrying the total bunch intensity. All bunches initially centred in the RF bucket are separated by a distance τ_{bb} and their position relative to the bucket centre are described by τ_i.

acceleration. This approximation is valid at the SPS flat top for a nominal bunch length of 1.65 ns. The behaviour of the bunch i is dictated by the equation

$$\ddot{\tau}_i + \omega_{s0}^2 \tau_i = \frac{\eta q}{\beta_s^2 E_s T_0} V_{ind}(\tau_i), \tag{5}$$

where V_{ind} is the voltage induced by the particles circulating ahead in the machine, T_0 the revolution period, q the charge of the bunch i, η the slip factor defined by $\delta T/T_0 = -\eta \delta p/p_s$ with δp the deviation from the synchronous momentum p_s, E_s the synchronous energy and $\beta_s c$ the velocity of the synchronous particle with c the speed of light. The SPS optics used in simulation (Q20) gives a slip factor $\eta \approx -3.1 \times 10^{-3}$. The synchrotron frequency $f_{s0} = \omega_{s0}/2\pi$ is defined by

$$\frac{\omega_{s0}^2}{\omega_0^2} = \frac{h V_{rf} \eta \cos\phi_s}{2\pi \beta_s^2 E_s}, \tag{6}$$

where V_{rf} is the amplitude of the RF voltage, h the harmonic number defined by $\omega_{rf} = h\omega_0$ and ϕ_s the phase of the synchronous particle with respect to the RF. For a longitudinal line density $\lambda(\tau)$, the general expression of the induced voltage is

$$V_{ind}(\tau) = -qN_b \int_{-\infty}^{\infty} \lambda(\tau') W(\tau - \tau') d\tau', \tag{7}$$

where N_b is the number of charges in the bunch. For point-like bunches the line density takes only discrete values. The wake function is expanded to linear order for small amplitude synchrotron oscillations. The zero order term can be discarded and only the first order term contributes to the dynamics. The equation of motion of the i^{th} bunch considering the interactions with all preceding neighbours is

$$\ddot{\tau}_i + \omega_{s0}^2 \tau_i = \kappa \sum_{k=0}^{i-1} W'[(i-k)\tau_{bb}](\tau_i - \tau_k), \tag{8}$$

with $\tau_{bb} = 25$ ns for an LHC beam and $\kappa = \frac{\eta q^2 N_b}{\beta_s^2 E_s T_0}$. For a resonator with $Q \gg 1$,

$$\frac{W'_{//}(\tau)}{2\alpha^2 R_{sh}} = -e^{-\alpha\tau} \sqrt{4 + \left(\frac{\bar{\omega}}{\alpha} - \frac{\alpha}{\bar{\omega}}\right)^2} \sin(\bar{\omega}\tau + 1/Q). \tag{9}$$

The induced voltage cancels perfectly every τ_{bb} if the resonant frequency is

$$f_r = \frac{k - \frac{1}{\pi Q}}{\tau_{bb}\sqrt{4 - 1/Q^2}}. \tag{10}$$

In the SPS case with the 630 MHz HOM, this corresponds to frequencies $f_r \approx k \times 20$ MHz for $k = 1, 2, 3...$ and therefore, if the resonant frequency of the mode is a multiple of 20 MHz, the growth rate of the instability is null. For a realistic bunch distribution, the bunch centre sees an induced voltage close to zero. This explains the improvement of the beam stability with realistic bunch trains for resonant frequencies of 620 MHz and 640 MHz. From Eq. (8), the growth time of the instability can be computed for the bunch i taking into account all preceding bunches,

$$\frac{1}{\Im(\omega)} = \left[\Im\left(\omega_{s0}^2 - \kappa \sum_{k=0}^{i-1} W'[(i-k)\tau_{bb}] \right)^{\frac{1}{2}} \right]^{-1}. \tag{11}$$

The expression (11) can be computed numerically for an intensity above threshold considering different number of bunches coupled, see Fig. 3. For the nearest neighbour interaction the growth time appears to be symmetric between the two 20 MHz lines. An asymmetry appears when the number of bunches coupled increases. After adding more than 10 bunches the growth time does not change significantly. This result is reasonable since the wake function decreases by a factor 2.7 over four bunches. However, compared to the realistic bunch case, it suggests that a frequency shift in the positive direction degrades beam stability. This analytical estimation is confirmed by simulations of 72 point-like bunches (Fig. 4).

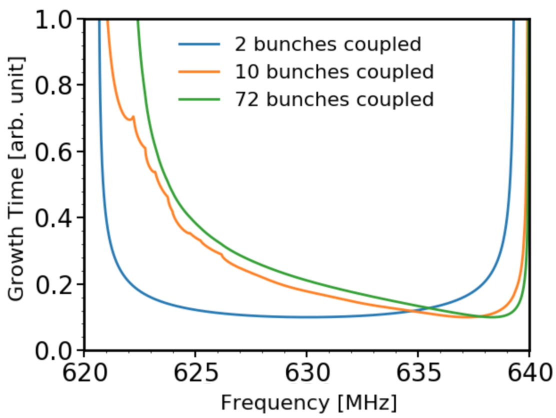

Figure 3: Instability growth time for a train of point-like bunches. An intensity above threshold is used and the sum in (11) is truncated for different lengths of interaction between bunches.

STABILITY OF REAL BUNCH TRAIN

With a realistic bunch distribution the induced voltage of each bunch is created over a finite length, which introduces a phase shift in the voltage overlap seen by the trailing bunches. The analytical bunch model can be extended by considering the induced voltage of a bunch with line density $\lambda(\tau)$ acting

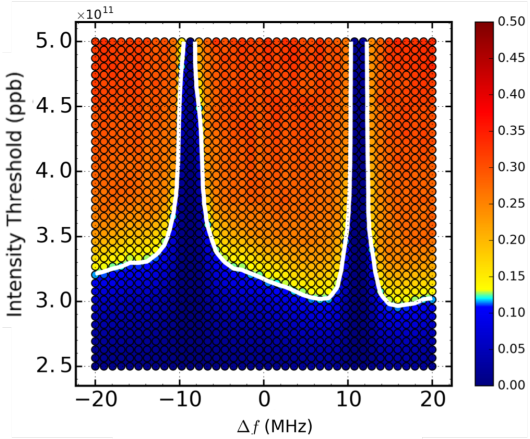

Figure 4: Intensity threshold simulated for 72 point-like bunches with 630 MHz HOM impedance only in a single 200 MHz RF system with a voltage of 10 MV. The resonant frequency is shifted from 630 MHz to 630 ± 20 MHz. Colours represent the maximum amplitude of the bunch position oscillations, normalised by the average one.

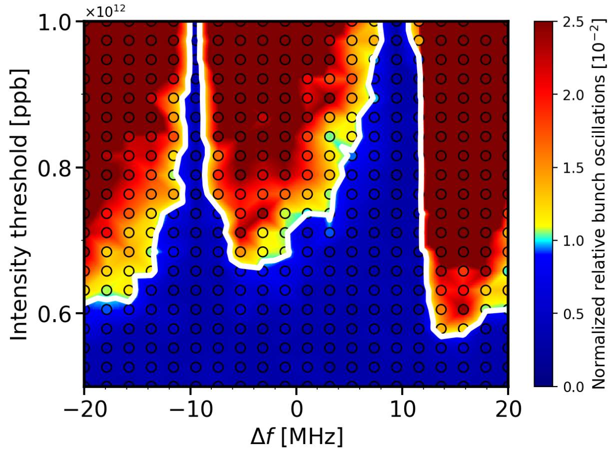

Figure 6: Intensity threshold simulated for 72 realistic bunches of length 1.65 ns with 630 MHz HOM impedance only. A single RF system at 200 MHz with a voltage of 10 MV is used. The resonant frequency is shifted from 630 MHz to 630 ± 20 MHz. Colours represent the maximum amplitude of the dipole oscillations of the last bunch in the train normalised by the average.

on the point-like bunches,

$$V_{ind}(\tau_{bb} + \Delta\tau) = -qN_b \int_{-\infty}^{+\infty} \lambda(\tau')W'(\tau_{bb} - \tau')d\tau'\Delta\tau. \tag{12}$$

In this case the instability growth time for the bunch i becomes

$$\frac{1}{\Im(\omega)} = \left[\Im \sqrt{\omega_{s0}^2 - \kappa \sum_{k=0}^{i-1} \int \lambda(\tau')W'[(i-k)\tau_{bb} - \tau']d\tau'} \right]^{-1} \tag{13}$$

Assuming a Gaussian bunch with a bunch length of 1.65 ns, the expression (13) can be calculated numerically for an intensity above threshold, see Fig. 5. The picture is similar

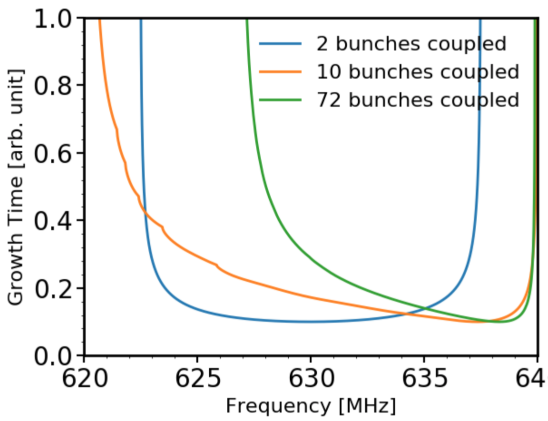

Figure 5: Instability growth time for a train of point-like bunches and an induced voltage generated by a Gaussian bunch of length 1.65 ns. An intensity above threshold is used and the sum in (13) is truncated for different lengths of interaction between bunches.

to the simpler point-like bunch model. The asymmetry between odd and even 20 MHz lines is comparable, indicating that synchrotron intra-bunch motion plays a significant role in determining stability threshold for a bunch train. Indeed, simulations at flat top with realistic bunches of 1.65 ns and the HOM impedance only (Fig. 6), exhibit the same asymmetry as the realistic SPS simulations (Fig. 1). The Figure 6 shows the threshold for the dipole oscillations but similar picture is obtained for the threshold of quadrupole oscillations. The stability improvement is the largest for resonant frequencies at 620 MHz and 640 MHz. At odd multiples of 20 MHz the symmetry is similar to the point-like bunch model but at even values the stability is reversed. These simulations indicate that the 630 MHz HOM is the main contributor to the instability and the interplay with other impedance sources does not necessarily has to be taken into account.

It should be noticed that even if a solution is found to modify the frequency of the HOM, in reality the possibilities of shifting the resonant frequency are limited. A frequency shift in the 640 MHz direction is beneficial for beam stability as long as the HOM damper can handle the increased heatload. On the other side, the region of increased stability toward the 620 MHz notch is very narrow (Fig. 6) and a sufficient frequency shift cannot be achieved by means of RF couplers [6]. Note that, if the bunch spacing is increased to 50 ns—solution considered if the LHC suffers e-cloud effects—the 630 MHz band becomes a region of higher stability.

CONCLUSION

After upgrading the SPS, the effect of the 630 MHz HOM on beam stability must be reduced to enable the bunch intensity required by the HL-LHC project. Further damping of the HOM through the use of RF couplers is difficult, but substantial improvement can be obtained by shifting its resonant frequency. The simulations indicate that the increase of intensity threshold, observed for a train of 72 bunches, is principally due to the change in HOM frequency. Similarly to the case of a ring filled with equally spaced bunches, if the resonant frequency overlaps a beam spectrum line or sits exactly between them, the bunches experience zero growth rate of instability. A shift of the mode frequency in the positive direction, toward the spectrum line at 640 MHz, is favourable for beam stability but increases the heat-load. On the contrary, a shift in the negative direction would have no detrimental effect on the heating but, it is observed, that the frequency band where the stability is improved is very narrow around 620 MHz. Since most of the mode energy is stored in the cavity volume, a sufficient detuning cannot be achieved by means of RF couplers. This mitigation scheme is therefore jeopardised and the efforts focus now on an improved damping scheme with an optimal positioning of new HOM couplers.

The point-like bunch model is able to account for the large gain in stability observed at resonant frequencies of 620 MHz and 640 MHz for bunch trains but has difficulties to reproduce some of the finer details observed with real bunches. Nevertheless, for odd values of 20 MHz harmonics, the threshold is similar to the point-like bunch model.

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