Possible Mitigations of Longitudinal Intensity Limitations for HL-LHC Beam in the CERN SPS

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I know of no time in human history where ignorance was better than knowledge.

— Neil Degrasse Tyson

À ma famille ...

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J. R.

Abstract

The Super Proton Synchrotron (SPS) at CERN is the injector of the Large Hadron Collider (LHC), the world's largest particle collider. The High-Luminosity LHC (HL-LHC) project is a major step forward in the improvement of the LHC performances and it requires a doubling of the nominal bunch intensity of the current LHC beam.

In the SPS, multi-bunch instabilities and particle losses limit the beam intensity that can be accelerated to 450 GeV/c and transferred to the LHC. Without mitigation measures, the bunch intensity threshold for longitudinal instabilities is three times below the nominal intensity of the LHC beam. Moreover, the present limited RF power is not sufficient to accelerate beams with intensities well above nominal without substantial particle losses and a reduction of the RF voltage available for the beam at the flat top energy.

The SPS will undergo significant upgrades but they may not be sufficient to ensure the stability of the HL-LHC beam. The objectives of this doctoral research are to study the longitudinal intensity limitations of the LHC proton beam in the SPS and to find possible mitigation measures to ensure the beam stability and quality at HL-LHC intensity.

Beam measurements and particle simulations are used in conjunction with analytical estimations to study the multi-bunch instabilities during the cycle in the SPS. This work attempts to identify the main sources of instabilities and beam quality degradation. Possible scenarios of mitigation measures are investigated to explore the future beam parameters achievable after upgrades. The effects on beam stability of the foreseen RF upgrade, the double RF operation and the reduction of various longitudinal beam-coupling impedances are analysed in detail. The scenario of a lower-harmonic RF system in the SPS, for particle losses reduction, is also studied.

Key words: Particle accelerators, longitudinal beam dynamics, CERN SPS, beam instability, double RF system, macroparticle simulations, coupled-bunch instability, high-intensity beams.

Résumé

Le Super Synchrotron à Protons (SPS) du CERN est l'injecteur du Grand Collisionneur de Hadrons (LHC), le plus grand collisionneur de particules au monde. Le projet LHC à haute luminosité (HL-LHC) constitue une avancée majeure dans l'amélioration des performances du LHC et nécessite un doublement de l'intensité nominale des paquets du faisceau LHC actuel.

Dans le SPS, les instabilités multi-paquets et les pertes de particules limitent l'intensité du faisceau qui peut être accélérée à 450 GeV/c et transférée au LHC. Sans mesures d'atténuation, le seuil d'intensité des paquets de l'instabilité longitudinale est trois fois inférieur à l'intensité nominale du faisceau du LHC. De plus, la puissance RF actuelle limitée ne suffit pas pour accélérer les faisceaux d'intensités supérieures à la valeur nominale sans des pertes substantielles de particules et une réduction de la tension RF disponible pour le faisceau à l'énergie supérieure.

Le SPS va subir d'importantes mises à niveau, mais celles-ci risquent de ne pas être suffisantes pour assurer la stabilité du faisceau HL-LHC. Les objectifs de cette recherche doctorale sont d'étudier les limites d'intensité longitudinale du faisceau de protons du LHC dans le SPS et de rechercher les mesures d'atténuation possibles pour assurer la stabilité et la qualité du faisceau à l'intensité HL-LHC.

Des mesures de faisceau et des simulations de particules sont utilisées conjointement avec des estimations analytiques pour étudier les instabilités multi-paquet pendant le cycle. Ce travail tente d'identifier les principales sources d'instabilités et de dégradation de la qualité du faisceau. Des scénarios possibles de mesures d'atténuation sont étudiés pour explorer les paramètres de faisceau futurs pouvant être atteints après les mises à niveau de la machine. Les effets sur la stabilité du faisceau de la mise à niveau RF prévue, le fonctionnement en système double RF et la réduction de diverses impédances de couplage longitudinal du faisceau sont analysés en détail. Le scénario d'un système RF à harmonique inférieure dans le SPS, pour la réduction des pertes, est également étudié.

Mots clefs : Accélérateurs de particules, dynamique longitudinale des faisceaux, CERN SPS, instabilité de faisceau, système à double RF, simulations de macro-particules, instabilité de paquets couplés, faisceaux à haute intensité.

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List of Frequently Used Symbols

A	Bucket area, longitudinal acceptance
₿, ₿	Magnetic induction field, value of the magnetic induction field
С	Speed of light
е	Elementary charge
Ē, E	Electrical field, value of the electrical field
Ε	Energy of an arbitrary particle
δE	Energy gain at every revolution period of an arbitrary particle
ΔE	Energy deviation of a particle with respect to the synchronous one
E_s	Energy of the synchronous particle
f_0	Revolution frequency
$f_{ m RF}$	Frequency of the RF system
f _r	Resonant frequency of an impedance
f_s	Synchrotron frequency
f_{s0}	Linear frequency of synchrotron oscillations
h	Harmonic number
h_{200} , h_{800}	Harmonic number of the two SPS RF systems at 200 MHz, 800 MHz
J	Action variable
J_n	Bessel function of the first kind and order n
K(x)	Complete elliptic integral of the first kind
n	Ratio of the harmonic number in a double RF system
N_b	Bunch intensity
$N_{\rm th}^{\rm CBI}$	Analytic estimation of the coupled-bunch instability threshold
$N_{\rm th}^{ m LLD}$	Analytic estimation of the loss of Landau damping threshold
p	Momentum
Q	Quality factor
q_p	Momentum filling factor

r	Voltage ratio between two RF systems in the double RF operation.
R_2	Series impedance of the travelling-wave structure
R _{sh}	Shunt impedance
V ₂₀₀ , V ₈₀₀	Amplitude of the voltage at 200 MHz and the voltage at 800 MHz respectively
v _g	Group velocity of the travelling-wave structure
V _{ind}	Induced voltage
Z_0	Line impedance of the travelling-wave structure
Z_b	Impedance of the travelling-wave structure seen by the beam
$Z_{\rm RF}$	Impedance of the travelling-wave structure seen by the generator
α	Momentum compaction factor
β	Reduced relativistic velocity
γ	Lorentz factor
ϵ	Bunch emittance
ϵ_{95}	Bunch emittance containing 95% of the particles
η	Slip factor
θ	Azimuthal phase position
τ	Bunch length
$\hat{ au}$	Synchrotron oscillation amplitude in time unit
$ au_{95}$	Bunch length containing 95% of the particles
$ au_{4\sigma}$	Bunch length from the FWHM of the bunch profile, rescaled to 4σ Gaussian
$ au_{bb}$	Spacing between bunches
$ au_{ m fil}$	Bunch length after filamentation (> 100 ms)
$ au_{ m FWHM}$	Bunch length from the FWHM of the bunch profile
$ au_{\max}$	Maximum bunch length in the batch
$ au_{ m min}$	Minimum bunch length in the batch
Δau	Amplitude of the bunch length oscillations
ϕ	RF phase coordinate
$\hat{\phi}$	Amplitude of phase oscillations in the longitudinal phase space
$\Delta \phi$	RF phase deviation with respect to the synchronous phase
$\phi_{ m slip}$	Phase slippage between the RF wave and the particle crossing the RF cavity
ϕ_{800}	Relative phase between the two SPS RF systems
ϕ_s	Synchronous phase
ϕ_{s0}	Synchronous phase in a single RF system
ψ	Angle variable
ω_0	Revolution angular frequency
$\omega_{ m RF}$	Angular frequency of the RF system
ω_r	Resonant angular frequency of the impedance
ω_s	Synchrotron angular frequency
ω_{s0}	Linear angular frequency of synchrotron oscillations
$\Delta \omega_s$	Synchrotron angular frequency spread

List of Abbreviations and Acronyms

AWAKE	Advances Proton Driven Plasma Wakefield Acceleration Experiment
BCT	Beam Current Transformer
BQM	Beam Quality Monitor
CERN	Organisation Européenne pour la Recherche Nucléaire
CNGS	CERN Neutrino to Grand Sasso
FF	Feedforward
FB	Flat Bottom
FT	Flat Top
DRF	Double RF system
HL-LHC	High-Luminosity Large Hadron Collider
HOM	Higher Order mode
LEP	Large Electron-Positron Collider
LHC	Large Hadron Collider
LINAC	Linear Accelerator
LIU	Large Hadron Collider Injectors Upgrade
LS2	Long Shutdown 2
OTFB	One-Turn-delay FeedBack
ppb	Particle per bunch
PS	Proton Synchrotron
PSB	Proton Synchrotron Booster
RF	Radio Frequency
SPS	Super Proton Synchrotron
SRF	Single RF system
TW	Travelling-Wave

List of Constants

Constant	Value	Quantity
с	299,792,458 m/s	speed of light
е	1.602 × 10 ⁻¹⁹ C	elementary charge

1 Introduction

Driven by curiosity to uncover the mysteries of the universe and wish to improve the knowledge in fundamental physics, large research establishments, like the Organisation Européenne pour la Recherche Nucléaire (CERN) in Switzerland, house particle accelerators of unprecedented scale. These high energy particle colliders are able to accelerate hundreds of billions of charged particles, all contained in bunches with a width (horizontally) on the µm scale. Particles are kept on well defined trajectories and travel distances several times the size of our own solar system, in a vacuum thinner than the interstellar space. Counter rotating beams of this type are then brought into collision at specified interaction points to study the subatomic interactions of matter in conditions never observed before on earth.

The high-energy particle accelerators represent only one percent of all the accelerators globally. However, there is a strive for constructing more powerful and reliable machines which is the driving force for a significant amount of researche in accelerator physics. The research conducted by facilities like CERN contributes greatly not only to the area of particle physics but also spur the development of cutting-edge technology with many applications far beyond the field itself and spin-off for society. For example, the evolution of superconducting cavities to accelerate the beam is now used in many biomedical applications to reduce the size of the installations and save operational costs. However, the superconducting technology is equally important in other fields, like the challenges of the energy production and transport in the future. Furthermore, particle accelerators are a tool widely deployed either in industrial applications (e.g., food products sterilization) or as a source of high energy particles to conduct research in material science (spectroscopy).

In the area of particle colliders, the Large Hadron Collider (LHC) at CERN is the largest one in the world housed in a 27 km long tunnel 100 m under the ground. As we seek to further understand particle physics, the LHC was designed to accelerate, in each of its two rings, a beam of protons containing 3.23×10^{14} particles to an energy of 7 TeV [1]. An energy of 6.5 TeV with an average number of protons per beam of 2.94×10^{14} has been achieved, which is limited by instabilities. The experiments performed during the collisions of these two beams led to the discovery of the Higgs Boson, announced on the 4th of July 2012. The Higgs boson is a

Chapter 1. Introduction

key component of the standard model of particle physics which was predicted in 1963 and this breakthrough led to the Nobel prize in the following year. The LHC can also accelerate and collide ion beams to study the behaviour of matter in a hot and dense state (quark-gluon plasma), close to the conditions present in the first moments of the cosmos.

The performance of the LHC has greatly improved over the years but one major step forward will be the High-Luminosity Large Hadron Collider project (HL-LHC) which requires significant upgrades across the entire accelerator complex [2]. The project aims at increasing the luminosity by means of, among others, increasing the bunch intensity N_b injected into the LHC ring, which is the number of particles in a bunch of the beam, normalized by the elementary charge, i.e. it has the unit particles per bunch (ppb). The luminosity is a measurement of the number of collisions given by the accelerator and is proportional to the square of the bunch intensity. The increase of intensity increases the instantaneous integrated luminosity, thus eventually allowing reducing the statistical uncertainties on the measurements by increasing the total number of collisions.

Before injection into the LHC, the beam is produced and transported in the injector chain made of smaller accelerators, accelerating the proton from their rest energy of 938.272 MeV/ c^2 to the LHC injection energy of 0.45 TeV. In order to achieve the goals of the HL-LHC project the beams supplied by the injector chain must also be improved. The LHC Injector Upgrade project (LIU) aims at identifying and removing the main limitations for HL-LHC beams [3].

The last accelerator in this chain is the Super Proton Synchrotron (SPS). The SPS is a machine which started operating in 1976 and has since gone far beyond the scope of its initial design. Therefore, it has limitations which are ultimately a bottleneck to the intensity that should be supplied to the HL-LHC. The objectives of this doctoral research are to study the intensity limitations in the longitudinal plane for the LHC proton beam in the SPS, and to find possible mitigation measures to ensure the beam stability and quality at HL-LHC intensity. Beam measurements and particle simulations are used in conjunction with analytical estimations to study the multi-bunch instabilities during cycle. This work attempts to identify the main sources of instabilities and beam quality degradation. In the following sections of this chapter, the type of accelerator studied, called synchrotron, will be introduced together with the CERN complex and the injector chain which produce the LHC proton beam. The present operation of the SPS and its limitations are introduced as well as the simulation code used to obtain the results presented below. The outline of the thesis concludes the chapter.

1.1 The Particle Colliders

Particle accelerators are machines designed to capture, confine and accelerate particles. Particle colliders are the same type of machine but, in addition, they store the beam to make it collide in various experiments or to produce secondary species at the fixed targets. A figure of merit of the accelerator is the beam energy or momentum, measured in the natural units of particle physics, the electron-volt (eV) or the eV/c respectively, with c the speed of light.

The electron-volt is defined as the amount of energy gained by an elementary charge in a difference of potential of one volt. The charged particles are accelerated by means of the electromagnetic force produced by a difference of electric potential in a gap. The energy increase in one crossing of the gap corresponds to the work of the force generated by the electric field, proportional to the gap length and the voltage. An electrostatic field (DC) is the most simple to produce since only two plates with opposite charges are needed which can create a maximum voltage of 10 MV with special designs like the Cockroft-Walton circuit or the Van de Graaff accelerator. However, this kind of field cannot be used in series or in a ring since Gauss's law shows that the total energy gained in a turn would be zero. To overcome this limitation an oscillating Radio Frequency (RF) field can be used. This was the suggestion of G. Ising in 1924 and it is the principle behind the Wideroe drift tube Linear Accelerator (LINAC) (1928) [4]. In order to reach always higher energy, the size of this type of linear accelerator becomes too large to be practical. If the trajectory of the particles is bent in a closed orbit, the same RF system can be reused to accelerate the beam at each passage. This is the main design principle of the synchrotron, the most common type of accelerator used today [5].

A synchrotron is a circular particle accelerator with a fixed radius. Its two main components are: the RF system, which confines particles in bunches and accelerates the beam, and a series of magnets to ensure the passage on the same path turn after turn, called a closed orbit. The advantage of a synchrotron is that the beam travels long distances in the same vacuum pipe only a few cm wide, which allows to reduce the size of the installation but imposes a synchronisation between the frequency of the RF field, $\omega_{\text{RF}}(p)$ and the magnetic induction field *B* generated by dipole magnets or main bends, i.e. the magnets that keep the particles on the orbit.

The RF system contains various control loops to correct deviations from the design parameters. These loops have a significant impact on the behaviour of the beam and their effects should be taken into account in the analysis of the longitudinal beam dynamics. This work concentrates on the RF system but it should not be forgotten that the machine consists also of many other components to allow beam injection and extraction, beam diagnostics, an ultra-high vacuum, and more.

Synchrotrons are used for the production of high energy beams and the CERN accelerator complex contains many of them of different sizes for various extraction energy demands, as introduced in the next chapter.

1.2 The CERN Accelerator Complex and the LHC Proton Beam

The CERN research center houses a large variety of accelerators built subsequently in the 65 years of the lab. The ones in operation today can be divided into two main families: linacs and synchrotrons. Many experiments take place along the chain of accelerators taking advantage of different beams at different energies (Fig. 1.1). To produce an LHC proton beam, four accelerators are involved, all affecting the characteristics of the beam. The Linear Accelerator



Figure 1.1 – Schematic view of the CERN accelerator complex (© CERN).

2 (Linac 2) has been the starting point of all proton bunches at CERN. It entered in operation in 1978 to provide an increased intensity to the experiments, and was shutdown in 2018. The Linac 2 accelerates particles extracted from a bottle of hydrogen to a kinetic energy of 50 MeV in an almost continuous beam. It will be replaced by a new 160 MeV Linac in 2020. The beam is then injected in the four rings of the Proton Synchrotron Booster (PSB) using a multi-turn injection process. By controlling the number of turns injected (which can be fractional), the total bunch intensity can be adjusted. The PSB accelerates the beam to an energy of 1.4 GeV. Six PSB bunches are injected into the Proton Synchrotron (PS) which uses different RF systems to split each of them into twelve to obtain the LHC proton batch containing 72 bunches spaced by τ_{bb} = 25 ns. These RF manipulations are also called RF gymnastics. Bunches produced in the 40 MHz RF system of the PS are too long to fit into the 200 MHz RF system of the SPS. Their length is reduced, before extraction, by bunch rotation in the longitudinal phase space, sharply increasing the voltage before extraction of the beam to the SPS when the bunch length is the shortest. The PS cycle is repeated four times to obtain, in the SPS, the four batches of the nominal LHC beam spaced by 200 ns with a total of 288 bunches. This beam is then accelerated in the SPS to an energy of 450 GeV and injected into the LHC.

At the nominal intensity of 1.15×10^{11} ppb, this scheme has been very efficient in delivering, for each LHC fill, the proton beams to each of the two rings of the LHC. However, the intensity increase required by the HL-LHC project poses serious challenges for all the injector chain. The maximum bunch length allowed for the LHC injection is fixed at 1.9 ns with an average value along the batches of 1.65 ns to be captured by the 400 MHz RF system of the LHC. Bunch stability is often most likely achieved by decreasing the density of particles in the bunch phase

Beam characteristics in SPS	LHC beam	HL-LHC beam	
Beam pattern	4×72		
Bunch spacing τ_{bb} [ns]	25		
Batch spacing [ns]	200		
Bunch length at extraction (4 σ) [ns]	1.65		
Intensity per bunch [ppb]	1.15×10^{11}	2.4×10^{11}	

Table 1.1 - Characteristics of the LHC and HL-LHC beams in the SPS.

space, thus having longer bunches. Because of the LHC 400 MHz RF system though, the maximum bunch length allowed for the LHC injection is fixed at 1.9 ns with an average value along the batches of 1.65 ns. The bunch length cannot therefore, be increased arbitrarily to improve beam stability. Significant hardware upgrades are necessary, and they will be implemented during the Long Shutdown 2 (LS2), starting in 2019. The characteristics of the LHC and HL-LHC beam in the SPS are summarised in Table 1.1 [3].

1.3 The Super Proton Synchrotron Present Operation

The Super Proton Synchrotron (SPS) is located approximately 40 m under ground in a 6.9 km long tunnel. It started operation in 1978, and has since accelerated many different beams including proton-antiproton, whose collisions led to the discovery of the W and the Z boson in 1983. From 1989 to 2000, the SPS operated as an electron and positron accelerator, supplying beam for the Large Electron-Positron Collider (LEP), accelerator housed in the same tunnel that is now the LHC's. Other examples are the protons beams used in the CERN Neutrino to Gran Sasso (CNGS) experiment which ended in 2012, proton bunches sent to the Advanced Proton Driven Plasma Wakefield Acceleration Experiment (AWAKE) and fixed target experiments in the North Area. Since 2008, the SPS is the injector of the LHC, accelerating proton and ion beams. The production of these beams is high-priority and demands short bunches at extraction with a high intensity.

During its long history, the SPS has gone far beyond its original scope, evolving to enable the acceleration of different species and ever increasing intensity. To reach the HL-LHC intensity goal many upgrades are necessary in the SPS and these are grouped under the LIU project. In its present and future configuration, the protons are injected at a momentum of approximately 26 GeV/c. Multiple batch injections during the 11.1 s long flat bottom allow accumulating the four batches of the LHC beam. Each batch is injected every 3.6 s and the remaining 300 ms before the start of the ramp provide sufficient time to complete the process of filamentation of the fourth batch. The beam is then accelerated in approximately 8 s reaching the magnetic flat top with a momentum of 450 GeV/c and it is extracted to the LHC after 500 ms, as shown in Fig. 1.2.

Already in present operation, LHC beam quality and stability in the SPS are challenging due to various effects. Large particle losses, increasing with intensity, are observed at the SPS



Figure 1.2 – Momentum program used in the SPS for the production of the LHC proton beam. Each of the four batches is injected every 3.6 s at flat bottom. The ramp starts at the cycle time t = 11.1 s and finishes at t = 19.53 s. The beam spends approximately half a second at flat top and is extracted at t = 19.93 s.

flat bottom [6] and multi-bunch longitudinal instabilities limit the bunch intensity [7]. The present SPS RF system is made of two four-section travelling-wave cavities (TWC) and two five-section TWC operating at 200 MHz that capture and accelerate the beam [8]. Two 800 MHz TWC support the main RF system to stabilise the beam [9]. Currently, to provide a good quality beam to the LHC, the second RF system at 800 MHz increases the synchrotron frequency spread inside the bunch and provides more effective Landau damping of beam instabilities [9]. In a single RF system (200 MHz), longitudinal instabilities appear for intensities three times lower than nominal due to the longitudinal beam-coupling impedance [10]. This impedance describes the electromagnetic field resulting from the interaction of the beam particles with their surrounding (vacuum pipe, cavities, etc.).

In the future, the SPS 200 MHz RF system will have more and shorter cavities, more power available and a better control of the beam loading through the new digital low-level RF control loops (LLRF) [11]. The parameters of both RF systems are presented in the Table 1.2 for the present and future (LIU upgrades) configurations. However, the SPS RF upgrade alone will not be sufficient to ensure beam stability at HL-LHC intensity. A minimum set of impedance reduction measures have been also included in the baseline SPS upgrades, which should on paper allow us to meet the HL-LHC target [7]. Further impedance reductions would be useful but are limited by feasibility and budget constraints. The mechanism producing the instability and its observation in the SPS is introduced in what follows.

Table 1.2 – Parameters of the two RF systems in the SPS. The harmonic number h corresponds to the amount of oscillations the RF wave makes in one revolution period T_0 of the particles in the ring. The maximum voltages are given before and after LIU RF upgrades. The 800 MHz RF system power plant have been upgraded already during the Long Shutdown 1 (LS1) in view of LIU.

	present	after RF upgrades	
Main RF system			
Harmonic number <i>h</i>	4620		
RF frequency f_{rf} [MHz]	200.222		
Maximum RF voltage V_{200}	7 MV	10 MV	
Second RF system			
Harmonic number <i>h</i>	18480		
RF frequency f_{rf} [MHz]	800.888		
Maximum RF voltage V ₈₀₀	2 MV	1.6 MV	

1.4 Beam Instabilities in the SPS

The circulating bunches induce an image current on the inner surface of the beam pipe with equal magnitude and opposite charge. Any discontinuity in the vacuum chambers leads to perturbations of the surrounding electromagnetic field and creates resistance (impedance) to the passage of the beam current. This impedance is the way to formalize the interaction of particles with their environment [12]. The electromagnetic perturbation trailing behind each beam particle, also called wake function, affects the bunch motion through the associated voltage [13].

As the beam intensity increases, the particles within the beam cannot be considered as a noninteracting single particles and collective effects become significant [14]. Beam instabilities belong to a wide range of collective effects in synchrotrons that have been the subject of intense research for several decades to push the machines performance further and further [5]. In the longitudinal plane, the induced voltage can change the bunch energy and it can induce coherent synchrotron oscillations. As it will be explained in the second chapter, depending on the magnitude of the vacuum chamber impedance and the bunch characteristics, this perturbation can be amplified and lead to beam instabilities which can be characterized by a threshold effect. For a given bunch distribution, the perturbation does not grow if the bunch intensity N_b is smaller than the threshold value, while instabilities do grow for intensities above this threshold. The induced voltage affects the bunch itself but can also propagate to the several trailing bunches leading to coupled-bunch instabilities.

Among all instabilities that can appear [12], the coupled-bunch instabilities (CBIs) are often the most severe limitations in both hadron and lepton synchrotrons operating at a high beam current. The lepton bunches are shorter and they profit from the synchrotron radiation damping, which is not the case for hadron bunches in most synchrotrons. To mitigate beam instabilities, passive and active damping methods can be used. The passive damping methods

Chapter 1. Introduction

rely on increased Landau damping, where a high-harmonic RF system is used as a Landau system and successfully suppress the instability at high bunch intensity. These systems are implemented at CERN in the SPS and successfully tested in the PS [15]. The active damping systems rely, commonly in synchrotrons, on signals from beam pick-ups to detect and control beam instabilities. These systems can proceed on a bunch-by-bunch basis using fast kicker magnets that correct the undesirable beam motion on a turn basis, or on a frequency-domain approach, which has the effect to reduce the impedance seen by the beam and causing the instability in a narrow band of frequencies [16]. These systems help to minimize the beam losses and provide a beam with reproducible parameters (intensity, bunch length, energy spread).

Presently in the SPS, the longitudinal multi-bunch instability is one of the most significant intensity limitations. In this thesis, instabilities are observed by means of evolving bunch profiles and losses on intensity measurements. The bunch profiles correspond to the instantaneous beam current measured, in the SPS, by a wall-current monitor [17] and it gives a measurement of the number of particles crossing the instrument at a given arrival time. The signal from the wall-current monitor is collected by an oscilloscope connected through a fiber-optic link. The bunch length can be extracted from bunch profiles during the cycle, and, as an unstable beam often features a strongly varying bunch length, it is a good indicator of the beam stability. An example of a multi-bunch instability onsetting at arrival at flat top is shown in Figure 1.3 for a batch of 12 bunches. The maximum and minimum bunch length along the bunch train deviate from the average with a large oscillation amplitude. It is observed that the amplitude of this oscillation is growing along the batch, the last bunch of the train being the most unstable. This is the signature of a coupled-bunch instability where subsequent bunches are more and more affected. This type of instability is typically due to narrowband resonant impedance with a wakefield propagating over several bunches. The intensity threshold $N_{\rm th}^{\rm CBI}$ of this coupled-bunch instability depends on the longitudinal emittance ϵ , the bunch length τ and the synchronous energy E_s . It can be calculated analytically for a given resonator and it scales like [19]

$$N_{\rm th}^{\rm CBI} \propto \frac{\epsilon^2}{E_s \tau}.$$
 (1.1)

Even though bunches becomes more rigid when the energy increases, they are more susceptible to be excited collectively. As it will be introduced in Section 2.8 and applied during the thesis, the coupled-bunch instability is sensitive to the synchrotron frequency spread, which, for a constant longitudinal emittance, decreases when the energy increases. The threshold is minimal at flat top energy.

The second important measurement is the losses on the intensity. The total intensity can be measured by a Beam Current Transformer (BCT). This device provides the total intensity inside the SPS, averaged over a turn, with a sampling of 5 ms. This intensity is also presented in Figure 1.3, divided by the number of bunches (12). Since this value is integrated over the ring, it contains also particles that are not properly captured by the RF system but still traveling inside the ring. This is called uncaptured beam.



Figure $1.3 - \text{Example of beam measurements for a batch of 12 bunches with a bunch intensity around <math>1.4 \times 10^{11}$ ppb at injection (above nominal) and the onset of instability. The average bunch length (blue curve), the min/max deviation of bunch length (orange) and the bunch intensity (green) during SPS cycle are shown. The dashed line (black) is the momentum program presented in Figure 1.2. The vertical dashed line in red corresponds to the onset of the instability. At the start of acceleration, the uncaptured beam is lost and the intensity reduces suddenly. The presence of uncaptured beam is due to the bunch distribution in phase space at injection, defined by the bunch rotation in the PS [18].

However, to investigate further the mechanism behind particle loss and instability, and find possible cures, particle tracking simulations are used in the following chapters. Analytical estimations can be used but they are often based on simplified models for a single RF system, and do not describe sufficiently a complicated machine like the SPS, in the double RF operation. To predict the beam behaviour in simulation, an accurate impedance model of the ring is necessary and this will be introduced in Chapter 3. Moreover, beam measurements in conditions close to the one of the HL-LHC beam cannot be achieved. The power of the present RF system is limited, in turn limiting the beam intensity that can presently be accelerated. Predictions of future performance and longitudinal instability thresholds rely mainly on numerical simulations.

1.5 Particle Tracking Simulations and the Code BLonD

The particle tracking simulation is a powerful tool in the analysis of the instability mechanisms of the long train of bunches in the double harmonic RF system interacting with the large number of elements in the ring. Developed at CERN, Beam LONgitudinal Dynamics (BLonD)

is a 2D particle tracking code, modelling the longitudinal phase space motion of single and multi-bunch beams in multi-harmonic RF systems [20]. The particle motion is simulated through a sequence of longitudinal energy kicks and drifts. The equations of longitudinal motion are discretised in time on a turn-by-turn basis with a time step equal to the revolution period $T_0 = 23.1 \,\mu s$ in the SPS. Collective effects are taken into account by computing, on a slicing of the bunch profiles, the induced voltage (added to the RF voltage) for a given impedance source and accumulated over several turns. Various beam control loops of the Low-Level RF (LLRF) system are tailor-made for each of the CERN synchrotrons; for example, the phase, frequency and synchro-loops, the one-turn delay feedback and the injection of RF phase noise, used for controlled emittance blow-up. The code is initially written in Python but the computationally intensive parts are in C++ [21]. It has been benchmarked against measurements in different CERN accelerators [6, 22, 23] and also against other simulations codes like PyOrbit [24], Headtail [25] and ESME [26]. The code has been proven to be reliable and is now used to study performance of rings, in longitudinal plane, at CERN and even outside the laboratory.

This simulation code BLonD is used in this thesis, to study the longitudinal beam stability in the SPS. There are several challenges in performing these simulations. The large number of bunches in the nominal LHC batch (288) makes simulations, for the full acceleration cycle (19.93 s), very demanding. Simulations are usually restricted to a single batch of 72 bunches (or less), since batches are weakly coupled by the SPS impedance sources [27]. Then, a wide variety of effects impacts the beam dynamics. For example, beam loading in the 200 MHz RF system, instabilities, or particle losses. The effect of space-charge is not negligible at injection energy and is always included in the simulations via an inductive impedance [28]. The initial particle distribution in phase space, defined by bunch rotation in the PS [29], has also to be taken into account for simulations at flat bottom. The double RF operation, the LLRF control loops and the controlled emittance blow-up can be also correctly included, if necessary. Finally, the complicated SPS impedance model (introduced in Chapter 3) requires careful convergence studies with simulation parameters. In the following chapters the results of simulations are benchmarked against beam measurements.

As part of the thesis, the code has been adapted for multi-bunch simulations. The largest computational time being spent in the kick, the drift and the slicing of the bunch profiles, the algorithms of these parts of the code have been optimized and multithreading solutions have been implemented to be used in high-performance-computing resources. The optimizations brought the simulation time of 72 bunches at flat top and 12 bunches during acceleration with 1 million particles per bunch from a few days to less than 10 hours. The optimized simulations have been carefully benchmarked against the previous version of the code and similar results are obtained. The necessary routines to monitor the beam parameters in multi-bunch (emittance, bunch length, synchrotron frequency distribution, bunch distribution in phase space) have also been implemented. It was an important step forward in the study of the LHC beam stability since simple analytic models do not fit the observations.

1.6 Thesis Outline

Major upgrades are required in the SPS to allow the production of the HL-LHC proton beam. The bunch intensity, extracted to the LHC after LIU upgrades should reach 2.3×10^{11} ppb with a loss budget, during the acceleration cycle, of 10%. At the present time, important particle losses and longitudinal collective effects restrict the bunch intensity that can be accelerated to flat top and the upgrades of the machine, already scheduled, represent the minimum requirement to fulfil the target of the LIU project.

The objectives of this doctoral research are to study the longitudinal intensity limitations of the LHC proton beam in the SPS and to find possible mitigation measures to ensure beam stability and quality at HL-LHC intensity. Beam measurements and particle simulations are used, supplemented by analytical estimations where possible. This work attempts to identify the main sources of instabilities and beam quality degradation. Possible scenarios of mitigation measures are investigated, to explore the future beam parameters achievable after LIU upgrades and to put requirements to the impedance reduction campaign. The effects on beam stability, of the foreseen RF upgrade, the double RF operation and of various longitudinal beam-coupling impedances are analysed in detail. The scenario of a lower-harmonic RF system, in the SPS, for reduction of capture losses, is also studied. The effects of various low-level RF loops are also included in simulations, which are done for single and double RF systems.

This PhD thesis is divided into five chapters, in addition to the introduction. In Chapter 2 the theoretical aspects of the longitudinal beam dynamics are discussed and important formulas are derived. In the third chapter the intensity limitations in SPS are studied. The beam loading limitation is analysed. The longitudinal beam-coupling impedance of the ring is also presented together with beam measurements of instability at SPS flat top (450 GeV/c). The cures presently implemented are also investigated. The mechanism of losses, at SPS flat bottom, is also introduced. It has been explored in beam measurements and particle tracking simulations of 72 bunches.

Chapter 4 is dedicated to the study of the beam instabilities, undertaken through beam measurements with batches of 12 bunches. The measurements were done in the single and the double RF system of the SPS. The effect of the different low-level RF controls of the 200 MHz RF system on the stability threshold are studied in a single RF system. This allowed to distinguished the low-level RF systems that have to be modelled in simulations. The one-turn-delay feedback of the cavities has the largest effect on the beam stability threshold during cycle. These studies also allowed to benchmark simulations using the complete longitudinal impedance model of the SPS. The instability thresholds simulated and measured at flat top show a sound agreement, which gives us good confidence in predictions of beam stability after the LIU upgrades. In the double RF system, studies of the voltage ratio between the two RF systems of the SPS allowed to obtain an optimal voltage program for the 800 MHz RF system to improve the beam stability during the acceleration cycle. This voltage program has been

successfully implemented and tested in operation and it is now used routinely.

In Chapter 5 the beam stability after LIU upgrades is investigated. The effect on beam stability of the RF upgrade is studied. The effect of various contributions to the longitudinal SPS impedance model is also evaluated to determine the most critical ones. The foreseen SPS impedance reduction campaign relies on these simulation results. Different scenarios for further impedance reduction are discussed. Possible ways of reducing the most critical impedance, for beam stability, are examined in details.

In the last chapter the particle losses and a possible cure are discussed. The scenario of a lower-harmonic RF system for bunch capture in the SPS as a loss mitigation scheme is studied in detail.

2 Longitudinal Beam Dynamics

In this chapter, the basics of the synchrotron motion and the electromagnetic interactions of particles with their surrounding, based on Refs. [5, 12–14, 30–33], are introduced, with focus on the key concepts necessary to understand the work developed in the following chapters.

In the first section, the single-particle equations of motion in the longitudinal plane are derived. The Hamiltonian formalism is introduced in Section 2.2 with the concept of the RF bucket, while in Section 2.3 the synchrotron frequency distribution of the particles in the longitudinal phase space is discussed for single and double RF systems. The interaction of the particles with their surrounding is treated later, first by introducing the concepts of wakefields and impedances in Section 2.4 and then by discussing the collective motion of the particles in terms of the Vlasov equation in Sections 2.5 and 2.6. The linearized Vlasov equation is used to calculate the beam transfer functions in Section 2.7 and finally the single and multi-bunch instability growth rates and thresholds are obtained in Section 2.8.

2.1 Single-Particle Motion

The motion of a particle with a charge *e* and a velocity \vec{v} in a synchrotron is determined by the Lorentz force

$$\vec{F} = e(\vec{\mathbb{E}} + \vec{v} \wedge \vec{\mathbb{B}}), \tag{2.1}$$

where \vec{E} is the electric field and \vec{B} is the magnetic induction field. In a synchrotron, the electric field accelerates the particles and the magnetic field constrains them on orbits along the ring. The two fields are orthogonal and the motion associated with each of them can be treated separately since the electric contribution to the force is much weaker than the magnetic contribution. The motion associated with the electric field is constrained in the direction tangential to the ring circumference and defines the longitudinal plane.

The momentum of the particle is $\vec{p} = \gamma m \vec{v}$, where $\gamma = 1/\sqrt{1-\beta^2}$ is the Lorentz factor, with velocity $v = \beta c$, *c* being the speed of light. The particles are accelerated (or decelerated) by an RF field, generated in RF cavities, and they follow closed orbits around the ring defined by the

magnetic fields.

In a synchrotron, the radius of the machine is fixed and the bending radius can only vary within the length of the beam pipe. For a fixed bending radius ρ_0 corresponding to the centre of the bending magnets, the circumference of the ring C_0 is given by

$$C_0 = 2\pi\rho_0 + L,$$
 (2.2)

where *L* is the total length of the straight sections that are usually included in the machine. An average radius R_0 can also be used such that $C_0 = 2\pi R_0$. The position of the particles in the ring can be described by using the polar coordinates (ρ , θ) as shown in Fig. 2.1 (the straight sections are not shown). It is convenient to define the motion with respect to a particle of reference which is synchronized with the RF voltage, i.e. its RF phase angle $\phi = \omega_{\text{RF}} t$ has the same value every time the particle crosses the RF cavity, where ω_{RF} is the angular RF frequency. For this reason, the reference particle is called the synchronous particle and its RF phase, ϕ_s is called the synchronous phase. This particle should follow exactly the design orbit of the machine with a bending radius ρ_0 , passing through the centre of all magnets.

The amplitude of the magnetic induction field *B* is constrained by the bending radius ρ_0 and



Figure 2.1 – Schematic view of a synchrotron with a bending radius ρ_0 . The two coordinate systems used to describe the motion are shown. The first, attached to the laboratory frame, is a polar coordinate system (ρ, θ) . The second, attached to the moving synchronous particle, is a Cartesian coordinate system $(\hat{x}, \hat{y}, \hat{s})$. A magnetic induction field \vec{B} is present along the ring circumference, pointing downward in the laboratory frame. An RF system provides an oscillating electric field \vec{E} in the longitudinal direction to accelerate the particles.

the momentum of the synchronous particle, p_s , by the relation [32]

$$B\rho_0 = \frac{p_s}{|e|},\tag{2.3}$$

where $B\rho_0$ is called the magnetic rigidity. The value of *B* is varied during acceleration to follow Eq. (2.3).

The synchronous particle moves at the angular velocity $\dot{\theta}_s = \omega_0$, its momentum is $p_s = \gamma m v_s$, where v_s is the velocity of the synchronous particle in the longitudinal direction, and its energy is $E_s = \sqrt{p_s^2 c^2 + m^2 c^4}$. The revolution period along the ring is

$$T_0 = \frac{C_0}{\beta c} = \frac{2\pi}{\omega_0}.$$
 (2.4)

Since the synchronous particle is synchronized with the RF field, the angular frequency of the RF system follows the relation

$$\omega_{RF} = h \times \omega_0, \tag{2.5}$$

where *h* is an integer called the harmonic number. This means that the RF phase ϕ_s of the synchronous particle is related to its azimuthal position θ_s by the equation

$$\phi_s = h\theta_s. \tag{2.6}$$

The motion of off-momentum particles (non-synchronous) can be described with respect to the synchronous one. A particle with momentum $p = p_s + \Delta p$, varying slightly from p_s (i.e. $\Delta p \ll p_s$), travels on a different orbit of circumference $C = C_0 + \Delta C$. This effect is described by the momentum compaction factor α through the relation [31]

$$\frac{\Delta C}{C_0} = \alpha \frac{\Delta p}{p_s}.$$
(2.7)

The value of α , which depends on the optics design, can be positive or negative. The revolution period of this particle differs from T_0 by ΔT and can be expressed as

$$\frac{\Delta T}{T_0} = \frac{\Delta C}{C_0} - \frac{\Delta \beta}{\beta}.$$
(2.8)

Since $\Delta p/p_s = \gamma^2 \Delta \beta/\beta$, using the relation (2.8), the relative change in the revolution period of this particle is related to $\Delta p/p_s$ by the equation

$$\frac{\Delta T}{T_0} = \left(\alpha - \frac{1}{\gamma^2}\right) \frac{\Delta p}{p_s}.$$
(2.9)

The slip factor η is defined as

$$\eta = \frac{1}{\gamma_{\rm tr}^2} - \frac{1}{\gamma^2},\tag{2.10}$$

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where the transition energy with $\gamma_{tr} = 1/\sqrt{\alpha}$ was introduced. Finally, Eq. (2.9) can be expressed as

$$\frac{\Delta T}{T_0} = \eta \frac{\Delta p}{p_s}.$$
(2.11)

Below the transition energy, η is negative and the revolution frequency increases when the particle accelerates. Above the transition energy, particles with momenta higher than p_s have lower revolution frequencies which delay their arrival time at the accelerating gap. On the contrary, particles with smaller momenta arrive earlier in the accelerating gap. At the transition energy, when $\gamma = \gamma_{tr}$, the slip factor goes to zero. The revolution period is the same for every momentum and the synchrotron motion is frozen.

To derive the equation of particle motion with respect to the synchronous particle, we define the RF phase and energy deviation, $\Delta \phi$ and ΔE respectively, which can be written

$$\begin{cases} \Delta \phi = \phi - \phi_s, \\ \Delta E = E - E_s, \end{cases}$$
(2.12)

where ϕ and *E* are respectively the RF phase and the energy of an arbitrary particle.

First Equation of Motion

According to Eq. (2.5), the phase ϕ is related to the revolution period deviation ΔT by

$$\Delta \phi = \omega_{\rm RF} \Delta T. \tag{2.13}$$

The combination of Eq. (2.11) and (2.13) gives

$$\frac{\Delta\phi}{T_0} = \frac{h\eta\omega_0}{\beta^2 E_s} \Delta E,\tag{2.14}$$

where the relation

$$\frac{\Delta p}{p} = \frac{1}{\beta^2} \frac{\Delta E}{E}$$
(2.15)

was used. Generally, during acceleration, ϕ_s varies very slowly with respect to the revolution period, i.e. $\dot{\phi}_s \ll \dot{\phi}$. This is the case in all the CERN synchrotrons and usually in most proton synchrotrons in operation. This allows writing the following relation

$$\frac{d}{dt}\Delta\phi = \frac{d}{dt}\phi.$$
(2.16)

Therefore, Eq. (2.14) becomes

$$\dot{\phi} = \frac{h\eta\omega_0}{\beta^2 E_s} \Delta E, \tag{2.17}$$

where the dot denotes the first derivative with respect to time. Equation (2.17) is the first equation of particle motion connecting change in phase with energy deviation.

Second Equation of Motion

A particle of charge *e* passing through a RF cavity gains an energy

$$\delta E = eV(\phi). \tag{2.18}$$

For a sinusoidal RF voltage with amplitude V_1

$$V(\phi) = V_1 \sin \phi. \tag{2.19}$$

Defining again the energy gain of the particle with respect to the synchronous one, we have

$$\delta E - \delta E_s = e \left[V(\phi) - V(\phi_s) \right]. \tag{2.20}$$

Assuming that *E* is a smooth function of time, one can write $\delta E/T = \dot{E}$. This approximation is usually justified since in most synchrotrons the cycle lasts from thousands to several millions of turns and particles gain a tiny amount of energy each time they cross the RF cavity. Therefore, Eq. (2.20) can be written

$$\frac{\dot{E}}{\omega} - \frac{\dot{E}_s}{\omega_0} = \frac{e}{2\pi} \left[V(\phi) - V(\phi_s) \right].$$
(2.21)

It can be further approximated [31] to

$$\frac{d}{dt}\left(\frac{\Delta E}{\omega_0}\right) = \frac{e}{2\pi} \left[V(\phi) - V(\phi_s)\right],\tag{2.22}$$

which is the second equation of motion, connecting the change in energy deviation with the RF voltage.

Synchrotron Oscillations and Phase Stability

Combining Eq. (2.17) and Eq. (2.22) one obtains the following second-order differential equation

$$\ddot{\phi} - \frac{eV_1 h\eta \omega_0^2}{2\pi\beta^2 E_s} \left[\sin\phi - \sin\phi_s\right] = 0, \qquad (2.23)$$

where the double dot denotes the second derivative with respect to time. For small difference in the RF phase $\Delta \phi$ with respect to ϕ_s , one can approximate the bracket in the above equation by

$$\sin\phi - \sin\phi_s \approx \Delta\phi \cos\phi_s. \tag{2.24}$$

Substituting the relation (2.24) into Eq. (2.23), one ends up with the equation of the harmonic oscillator

$$\Delta \ddot{\phi} - \omega_{s0}^2 \Delta \phi = 0, \qquad (2.25)$$

where ω_{s0} is the linear angular frequency of synchrotron oscillations defined by

$$\omega_{s0} = \omega_0 \sqrt{-\frac{heV_1\eta\cos\phi_s}{2\pi\beta^2 E_s}}.$$
(2.26)

The stability is ensured only if ω_{s0} is real or equivalently if $\omega_{s0}^2 \ge 0$. The condition of phase stability is therefore

$$\eta \cos \phi_s < 0. \tag{2.27}$$

According to Eq. (2.10), η is negative below the transition energy and positive above, then the condition (2.27) is equivalent to

$$\begin{cases} 0 \le \phi_s < \pi/2 & \text{if } \gamma < \gamma_{\text{tr}} \text{ (below transition),} \\ \pi/2 < \phi_s \le \pi & \text{if } \gamma > \gamma_{\text{tr}} \text{ (above transition).} \end{cases}$$
(2.28)

In the SPS ($\gamma_{tr} \approx 18$, see Section 3.4.1), the LHC proton beams are injected above transition (26 GeV/c) and η is assumed to be positive in what follows.

The solution of Eq. (2.25) is

$$\phi(t) = \phi_s + \hat{\phi} \cos(\omega_{s0} t), \qquad (2.29)$$

where $\hat{\phi}$ is the amplitude of the phase oscillations. In the vicinity of the synchronous particle, the particle has a harmonic motion around the synchronous phase ϕ_s . When $\phi_s = \pi$, the energy gain at each revolution period from Eq. (2.18) is zero and the synchronous particle is not accelerated. The same apply below transition with $\phi_s = 0$. If $\phi_s < \pi$ and follows the conditions (2.28), the synchronous particle is accelerated.

2.2 Hamiltonian Formulation and RF Bucket

Using the second order equation of motion (2.23) one can write

$$\frac{d}{dt} \left[\frac{\dot{\phi}^2}{2} - \frac{\omega_{s0}^2}{\cos\phi_s} \frac{2\pi}{eV_1} U(\phi) \right] = 0,$$
(2.30)

where Eq. (2.26) was used and $U(\phi)$ is the potential defined by

$$U(\phi) = -\frac{e}{2\pi} \int_{\phi_s}^{\phi} \left[V(\phi') - V(\phi_s) \right] d\phi'.$$
(2.31)

Using the first equation of motion (2.17), one obtains a first integral of motion

$$H\left(\phi, \frac{\Delta E}{\omega_0}\right) = \frac{h\eta\omega_0^2}{2\beta^2 E_s} \left(\frac{\Delta E}{\omega_0}\right)^2 + U(\phi).$$
(2.32)

The function $H(\phi, \Delta E/\omega_0)$ is the Hamiltonian of the system. The Hamiltonian represents the total energy of the system, independent of the coordinate system up to some constant, and it
is a constant of motion for conservative systems.

The equations of motion (2.17) and (2.22), are the canonical equations of Hamilton for the two variables $(\phi, \Delta E/\omega_0)$,

$$\begin{cases} \frac{d}{dt}\phi = \frac{\partial H}{\partial\left(\frac{\Delta E}{\omega_0}\right)} = \frac{h\eta\omega_0}{\beta^2 E_s}\Delta E, \\ \frac{d}{dt}\left(\frac{\Delta E}{\omega_0}\right) = -\frac{\partial H}{\partial\phi} = \frac{e}{2\pi}\left[V(\phi) - V(\phi_s)\right]. \end{cases}$$
(2.33)

Note that Eqs. (2.32) and (2.33) are valid for an arbitrary RF voltage function.

The Hamiltonian (2.32) characterizes trajectories of constant energy which can be obtained from the following relation

$$\frac{\Delta E}{\omega_0}(\phi) = \pm \sqrt{\frac{2\beta^2 E_s}{h\eta\omega_0^2} \left[H - U(\phi)\right]}.$$
(2.34)

For the sinusoidal RF voltage of amplitude V_1 defined in Eq. (2.19), the potential well is

$$U(\phi) = \frac{eV_1}{2\pi} \left[\cos\phi - \cos\phi_s + (\phi - \phi_s)\sin\phi_s \right].$$
(2.35)

An arbitrary particle with an initial energy $H = H_p$ oscillates within the potential well, centred at ϕ_s .

There is a value of the Hamiltonian, $H_p = H_{sep}$, such that for particles with an energy below H_{sep} the motion is bounded within the potential well and unbounded for all energies above. The trajectory in phase space defined by H_{sep} is called a separatrix, which is the boundary of the RF bucket.

The separatrix has two turning points where $\Delta E = 0$. The first one is the unstable fixed point of the Hamiltonian function $(\pi - \phi_s, 0)$, where $U'(\phi) = 0$, and the second is the point $(\phi_u, 0)$ defined by the relation

$$H_{\rm sep} = U(\pi - \phi_s) = U(\phi_u).$$
 (2.36)

The case $\phi_s = \pi$ corresponds to a non-accelerating (stationary) bucket and when $\phi_s < \pi$, the RF system accelerates particles. For illustration, the RF voltage (top), the corresponding potential well (middle), and the trajectories in the longitudinal phase space (bottom) are plotted in Fig. 2.2 for the cases with $\phi_s = \pi$ (a) and $\phi_s = 0.9\pi$ (b). The latter value of ϕ_s is an example corresponding to the middle of the LHC acceleration cycle in the SPS.

The area enclosed by the separatrix is called the bucket area (or the longitudinal acceptance) and is usually measured in the unit of eVs. The value of the bucket area *A* is given by

$$A = \frac{1}{h} \oint_{\text{separatrix}} \frac{\Delta E}{\omega_0} d\phi.$$
 (2.37)



Figure 2.2 – RF voltage, potential well and particle trajectories in the phase space ($\phi, \Delta E/\omega_0$), above transition for the cases $\phi_s = \pi$ (a) and $\phi = 0.9\pi$ (b). The vertical dashed lines indicate the synchronous phase and the boundary of the separatrix.

Using the relation (2.34), the acceptance can be computed as

$$A = \frac{2}{h} \int_{\pi - \phi_s}^{\phi_u} \sqrt{\frac{2\beta^2 E_s}{h\eta\omega_0^2}} \left[H_{\text{sep}} - U(\phi) \right] d\phi.$$
(2.38)

The acceptance (2.38) gives the maximum area which particles can occupy in the longitudinal phase space. The particles within a RF bucket are called a bunch, which is not necessarily occupying the maximum area. All particles outside the separatrix are uncaptured, drift away from the bucket and are usually lost (can be recaptured) during acceleration. Within the RF bucket, the particles oscillate with a frequency depending on their position. The distribution of this synchrotron frequency within the RF bucket is considered in the next section.

2.3 Synchrotron Frequency Distribution

For particles with a small amplitude of oscillations, the synchrotron frequency in a single RF system (Eq. (2.26)) does not dependent on the oscillation amplitude. For large amplitude of oscillations, the non-linearity of the RF voltage is not negligible. For a given Hamiltonian H_p , which corresponds to a trajectory in phase space with maximum phase excursions ϕ_1 and ϕ_2 , such that $H_p = U(\phi_1) = U(\phi_2)$, the period of the synchrotron motion can be evaluated from Eq. (2.17) as

$$T_{s}(H_{p}) = \frac{2\beta^{2}E_{s}}{h\eta\omega_{0}^{2}} \int_{\phi_{1}(H_{p})}^{\phi_{2}(H_{p})} \left[\frac{\Delta E}{\omega_{0}}(\phi, H_{p})\right]^{-1} d\phi.$$
(2.39)

The synchrotron frequency on the trajectory defined by H_p is $f_s(H_p) = 1/T_s(H_p)$.

In a stationary bucket ($\phi_s = 0, \pi$) the maximum phase excursions are symmetric with respect to ϕ_s and can be written as $\phi_1 = \phi_s - \hat{\phi}$ and $\phi_2 = \phi_s + \hat{\phi}$, where $\hat{\phi}$ is the amplitude of the phase oscillations along the trajectory defined by H_p . Above transition, in a single RF system, the expression (2.34) can be used and the synchrotron angular frequency is

$$\omega_s(\hat{\phi}) = \omega_{s0} \frac{\pi}{2K\left(\sin\frac{\hat{\phi}}{2}\right)},\tag{2.40}$$

where

$$K(x) = \int_0^{\pi/2} \frac{du}{\sqrt{1 - x^2 \sin^2 u}}$$
(2.41)

is the complete elliptic integral of the first kind. For small amplitude oscillations, the equation (2.40) can be approximated by

$$\omega_s(\hat{\phi}) \approx \omega_{s0} \left(1 - \frac{\hat{\phi}^2}{16} \right), \tag{2.42}$$

where the synchrotron frequency is equal to ω_{s0} in the bunch centre. Figure 2.3 shows the synchrotron frequency distribution as a function of the phase amplitude $\hat{\phi}$ from Eq. (2.40) and the approximation given by Eq. (2.42). For an arbitrary $\hat{\phi}$ close to the synchronous phase, the



Figure 2.3 – Synchrotron frequency distribution in a single RF system as a function of the phase amplitude of the particle $\hat{\phi}$ (solid line) and its approximation given by Eq. (2.42) (dashed line).

spread of the synchrotron frequency between 0 and $\hat{\phi}$ defined by

$$\Delta\omega_s(\hat{\phi}) = \omega_{s0} - \omega_s(\hat{\phi}), \qquad (2.43)$$

is proportional to the square of the amplitude of the phase oscillations, i.e.

$$\frac{\Delta\omega_s}{\omega_{s0}} = \frac{\hat{\phi}^2}{16}.\tag{2.44}$$

The synchrotron frequency goes to zero when the phase amplitude approaches the separatrix.

The synchrotron frequency distribution is crucial for beam stability since it provides a natural stabilization mechanism called Landau damping. A simplified vision of this mechanism can be formulated as follows. A beam is made of many particles which can be considered as oscillators with frequencies $\omega_i = \omega_s(\hat{\phi}_i)$. For an external excitation F(t), which resonates with ω_i , the energy of this excitation can be shared between particles with a synchrotron frequency in the vicinity of ω_i . This could prevent the instability to grow. A detailed description can be found, for example, in Refs [12, 34].

An increase of the synchrotron frequency spread within the bunch provides, in general, more efficient Landau damping of beam instabilities [35, 36]. A possible way to increase the synchrotron frequency spread within the bunch is to increase its emittance or to use a second RF system with a higher harmonic number [9].

The second RF system, with voltage V_2 , increases the non-linearity of the RF wave and modifies the synchrotron frequency distribution. In a double RF system, the total voltage is given by

the following expression,

$$V_{\rm RF} = V_1 \left[\sin \phi + r \sin(n\phi + \Phi_2) \right], \qquad (2.45)$$

where $r = V_2/V_1$ is the voltage ratio, $n = h_2/h_1$ is the ratio of the harmonic numbers and Φ_2 is the relative phase between the two RF systems. In this case, the potential well is

$$U(\phi) = \frac{eV_1}{2\pi} \left\{ \cos\phi - \cos\phi_s + \frac{r}{n} \left[\cos(n\phi + \Phi_2) - \cos(n\phi_s + \Phi_2) \right] + (\phi - \phi_s) \left[\sin\phi_s + r\sin(n\phi_s + \Phi_2) \right] \right\}.$$
(2.46)

The relative phase Φ_2 has a big impact on the synchrotron frequency distribution and can be determined to maximize the synchrotron frequency spread in the bunch centre.

The zero-amplitude synchrotron angular frequency ω_{s0} is modified by the second RF system according to the expression [9]

$$\omega_s^2(\hat{\phi}=0) = \frac{\omega_{s0}^2}{\cos\phi_{s0}} [\cos\phi_s + r\,n\cos(n\phi_s + \Phi_2)]. \tag{2.47}$$

For a given ϕ_s the change of the synchrotron frequency in the bunch centre is maximum when

$$n\phi_s + \Phi_2 = 0, \pi. \tag{2.48}$$

At a given time in the cycle, the synchronous phase in a single RF system, ϕ_{s0} , is linked to the energy gain of the synchronous particle δE_s by Eq. (2.18) and is

$$\delta E_s = eV_1 \sin \phi_{s0}. \tag{2.49}$$

For the same energy gain δE_s , the synchronous phase ϕ_s in a double RF system is related to ϕ_{s0} by

$$\sin\phi_{s0} = \sin\phi_s + r\sin(n\phi_s + \Phi_2). \tag{2.50}$$

Applying condition (2.48) to Eq. (2.50), one obtains $\phi_{s0} = \phi_s$.

In a non-accelerating bucket above transition, the value of Φ_2 is either 0, called the bunchlengthening mode (BLM), or π , called the bunch-shortening mode (BSM). The names come from the effect these two modes have on the bunch length for n = 2. The case of the SPS with n = 4 is treated in more details in Section 3.4.2.

The value of *r* is also restricted. Indeed, if rn > 1 in Eq. (2.47), then $\omega_s^2(\hat{\phi} = 0) < 0$ for some regions and the potential well can have three minima instead of one, as shown for the case of the bunch-shortening mode in Fig. 2.4. In bunch-lengthening mode, local minima are created close to the centre of the potential well. This is the case for n = 4 as well. Thus, to avoid higher frequency buckets inside the main bucket, we choose to limit the voltage ratio between the two RF systems to

$$r \le \frac{1}{n}.\tag{2.51}$$



Figure 2.4 – Potential well defined in Eq. (2.46) in bunch-shortening mode with n = 2 and r = 0.25 (green), 0.5 (orange), and 0.75 (red).

The synchrotron frequency distribution can also be computed, using the action-angle variables defined below. The pair of canonical variables $(\phi, \Delta E/\omega_0)$ can be transformed into another set of canonical variables (\mathcal{J}, ψ) and the action variable \mathcal{J} can be defined as

$$\mathscr{J} = \frac{1}{2\pi} \oint \frac{\Delta E}{\omega_0} d\phi, \qquad (2.52)$$

where the integration is taken on particle trajectories in phase space. For conservative systems (as the one discussed here), \mathscr{J} is a constant of motion. The angle variable ψ corresponds to the fraction of the area the particle spanned during its motion on the ellipse defined by \mathscr{J} and increases by 2π in one synchrotron period. In the absence of perturbation, the equations of motion (2.17), (2.22) in the set of variables (\mathscr{J}, ψ) are

$$\begin{cases} \dot{\psi} = \omega_s(\mathcal{J}), \\ \dot{\mathcal{J}} = 0, \end{cases}$$
(2.53)

where $\omega_s(\mathcal{J})$ is the synchrotron frequency as a function of the action. Its value is obtained using the relation (2.33)

$$\omega_s(\mathscr{J}) = \frac{\partial H(\mathscr{J})}{\partial \mathscr{J}}.$$
(2.54)

Equation (2.54) is used below to compute numerically the synchrotron frequency distributions. The action-angle variables are also used to describe the collective motion of particles.

Examples of synchrotron frequency distribution for a single RF system and four possible double RF scenarios are shown in Fig. 2.5. In the BLM, the synchrotron frequency is reduced in the bunch centre, whereas in the BSM, the central synchrotron frequency is increased. For higher harmonic number, less voltage is needed to obtain the same synchrotron frequency spread



Figure 2.5 – Synchrotron frequency distribution in a single RF (black curve) and a double RF system above transition in the bunch-shortening mode with $\phi_2 = \pi$ (orange) and the bunch-lengthening mode with $\phi_2 = 0$ (blue) for two different harmonic ratios, n = 2 and n = 4. The voltage ratio is fixed to r = 1/n.

in the bunch centre. However, in the case of n = 4, the synchrotron frequency distribution exhibits a flat portion ($\omega'_s = 0$) and the spread increases for shorter emittance as compared to the case of n = 2. This flat region may also lead to a loss of Landau damping for long bunches as it will be explained in Section 2.7. The instabilities can be then triggered by any perturbation which comes in general under the form of a wakefield, studied in the next section.

2.4 Wakefield and Impedance

The electric field carried by charged particles circulating in the ring is perturbed by discontinuities of the beam pipe due to many elements of the machine; for example, vacuum flanges, pumping ports, RF cavities, beam measurement devices, etc. [37]. Due to the high proton beam energy in the SPS, in what follows, particles are considered to be relativistic, i.e. $\beta \sim 1$. In this case, the electric field generated by particles is radial to the direction of their motion with an open angle of about $1/\gamma$ [12]. The fields end at the beam pipe where an image current of opposite charge travels. The movement of the image charges is delayed by the discontinuities, which generate perturbations in the trailing electromagnetic field [13]. This field affects the particle itself and the trailing particles. In what follows, it is assumed that the particles are moving with the speed of light (high-energy particles) and therefore causality implies that there is no electromagnetic field in front of it, reason why these fields are known as wakefields [12, 13].

The wake function is the Green's function of the Maxwell's equations. Consider a source particle with charge *e* travelling with a velocity v = c inside a vacuum chamber such that its position is z = ct. This particle generates an electric field $\mathbb{E}(z, t)$ along the direction of the particle motion (longitudinal). A witness particle, following the source particle at a distance Δz behind the source particle and travelling at the same speed, sees an electric field $\mathbb{E}(z + \Delta z, t)$ generated by the source particle. The wake function can be written

$$W(\Delta z) = \frac{1}{e} \int \mathbb{E}(z + \Delta z, t) dz, \qquad (2.55)$$

where the integration domain is the length of the corresponding element and $W(\Delta z > 0) = 0$. The wake function is illustrated in Fig. 2.6. By virtue of the superposition principle and the 1-kick approximation, one can compute the Green's function of every element of the ring separately and then sum them [12].

The interaction between particles and their surrounding can also be described using the concept of coupling impedance. Applying a Fourier transform over the variable

$$\Delta t = \frac{\Delta z}{\beta c} = \frac{\phi}{\omega_{\rm RF}},\tag{2.56}$$

one gets the longitudinal coupling impedance [37]

$$Z(\omega) = \int W(\Delta t) e^{i\omega\Delta t} d\Delta t.$$
(2.57)

The impedance is a complex Hermitian quantity with $\text{Re}Z(\omega)$ and $\text{Im}Z(\omega)$ which are even and odd functions of ω , respectively. The wakefield can be expressed in terms of the impedance by using the inverse Fourier transform,

$$W(\Delta t) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} Z(\omega) e^{-i\omega\Delta t} d\omega.$$
(2.58)



Figure 2.6 – Schematic view of the wakefield generated by a source particle and seen by the witness particle. The particles travel with the same velocity v = c in the longitudinal direction, at a distance Δz from each other.

To compute the total wakefield, Maxwell's equations must be solved with boundary conditions for each element of the ring. The solution is generally obtained through numerical solvers as CST Particle Studio[®] [38], but analytical expressions can be derived for simplified models, see e.g. Refs. [12, 14, 37].

In many practical cases, the impedance can be described by the resonator model. In cavity-like objects, the electromagnetic behaviour can be modelled by a parallel RLC circuit defined by its resonant angular frequency ω_r , shunt impedance R_{sh} , and quality factor Q with an impedance which, for the longitudinal plane, is expressed as follows [37]

$$Z(\omega) = \frac{R_{\rm sh}}{1 + iQ(\frac{\omega}{\omega_r} - \frac{\omega_r}{\omega})}.$$
(2.59)

The corresponding wakefield is

$$W(\Delta t) = \begin{cases} 0, & \text{if } \Delta t < 0, \\ aR_{\text{sh}}, & \text{if } \Delta t = 0, \\ 2aR_{\text{sh}}e^{-a\Delta t} \left[\cos(\bar{\omega}\Delta t) - \frac{a}{\bar{\omega}}\sin(\bar{\omega}\Delta t)\right], & \text{if } \Delta t > 0, \end{cases}$$
(2.60)

with the parameters

$$a = \frac{\omega_r}{2Q}$$
 and $\bar{\omega} = \omega_r \sqrt{1 - \frac{1}{4Q^2}}$. (2.61)

Notice that each particle is affected by its own wakefield since $W(0) \neq 0$; this us a consequence of the beam-loading theorem [13]. The decay time of this resonant wakefield is given by

$$\frac{1}{a} = \frac{2Q}{\omega_r}.$$
(2.62)

This quantity gives an indication of how far the trailing particles can be influenced by the wakefield of the source particle, possibly over many buckets.

2.5 Multi-Particle Motion

Intensity effects included, the total voltage experienced by an arbitrary particle at RF phase ϕ along the accelerator can be expressed as the sum of the contributions from the RF field and the induced field,

$$V(\phi) = V_{\rm RF}(\phi) + V_{\rm ind}(\phi). \tag{2.63}$$

The induced voltage can be calculated by summing all wakefields generated by all particles circulating ahead of the particle at RF phase ϕ and the equation of motion becomes

$$\ddot{\phi} + \frac{\omega_{s0}^2}{V_1 \cos \phi_{s0}} V_{\rm RF}(\phi) = -\frac{\omega_{s0}^2}{V_1 \cos \phi_{s0}} V_{\rm ind}(\phi).$$
(2.64)

Once perturbations of the electromagnetic field are added, the single-particle equations of motion couple, more or less strongly, all the particles circulating in the ring. The potential well is affected, which modifies the synchrotron frequency distribution as well as the synchronous phase.

The number of particles in a proton bunch in the SPS is typically of the order of 10^{11} ppb with a number of bunches up to 288 for the LHC beam. The induced voltage is the result of a collective effect involving all particles in the ring. The computation of the induced voltage can be tackled by two approaches [12]. The first is the "particle approach" where the Hamiltonian function has to be modified to include the 10^{11} particles in the ring and the corresponding coupled equations of motion defined by Eq. (2.64) must be solved, typically in the time domain. This is the method which is usually implemented in the numerical simulations, as well as in the code BLonD , used in this thesis [20]. However, we are interested in macroscopic quantities like the bunch emittance and the evolution of the particle distribution. The microscopic motion, with wavelengths of the order of the separation between particles, does not give any insight into the bunch dynamics, and only the collective modes of oscillations can be observed in beam measurements. The mathematical description of these collective modes is based on the particle distribution function in phase space $\mathscr{F}(\phi, \Delta E/\omega_0)$. The distribution is normalised to unity such that

$$1 = \int \int \mathscr{F}\left(\phi, \frac{\Delta E}{\omega_0}\right) d\phi \, d\left(\frac{\Delta E}{\omega_0}\right),\tag{2.65}$$

where the integrals are taken over the area in phase space occupied by the bunch. The bunch profile (or line density) is the projection of the distribution function on the phase axis, i.e.

$$\lambda(\phi) = \int \mathscr{F}\left(\phi, \frac{\Delta E}{\omega_0}\right) d\left(\frac{\Delta E}{\omega_0}\right).$$
(2.66)

To quantify the size of a bunch in phase space, the bunch emittance

$$\epsilon = \oint_{\text{bunch}} \left(\frac{\Delta E}{\omega_0}\right) d\phi, \qquad (2.67)$$

is used, where the integral is taken on the outermost trajectory the bunch occupies in phase space, which corresponds to a Hamiltonian H_b . Notice that to measure the emittance in eVs, Eq. (2.67) must be devided by the harmonic number h. The emittance is related to the action variable defined in Eq. (2.52) by

$$\epsilon(H_b) = 2\pi \mathscr{J}(H_b). \tag{2.68}$$

From the Liouville's theorem, in a conservative system, the bunch emittance is a constant of motion. If the synchrotron motion is adiabatic, a change of emittance indicates that the system is gaining or losing energy from an external source, and possibly the presence of an instability. The condition for an adiabatic synchrotron motion can be expressed as follows [31]

$$\left. \frac{\dot{\omega}_{s0}}{\omega_{s0}^2} \right| \ll 1. \tag{2.69}$$

To have an adiabatic synchrotron motion, the rate of change of the synchrotron frequency must be small compared to the synchrotron frequency itself, which means that parameters of the Hamiltonian change slowly so that the particle trajectories remain iso-Hamiltonian in phase space. The trajectory is bounded by two phases ϕ_1 and ϕ_2 , where $H_b = U(\phi_1) = U(\phi_2)$ and the emittance can be also written as

$$\epsilon = 2 \int_{\phi_1}^{\phi_2} \sqrt{\frac{2\beta^2 E_s}{h\eta\omega_0^2} \left[U(\phi_1) - U(\phi) \right]} d\phi.$$
(2.70)

The formula (2.70) is used, in this thesis, to compute the maximum bunch emittance in particle simulations and beam measurements. The values of ϕ_1 and ϕ_2 also determine the corresponding bunch length $\tau = (\phi_2 - \phi_1)/\omega_{\rm RF}$. In beam measurements, it is found from the bunch profile, which leads to some freedom in the way the emittance can be computed. For example, the bunch length can be obtained from the Full Width at Half Maximum (FWHM) value of the bunch profile or a Gaussian fit.

Usually in the SPS, the convention is to use the FWHM bunch length, τ_{FWHM} , rescaled to 4σ assuming a Gaussian distribution, $\tau_{4\sigma}$, using the relation

$$\tau_{4\sigma} = \frac{2}{\sqrt{2\ln 2}} \tau_{\rm FWHM},\tag{2.71}$$

so that $(\phi_2 - \phi_1)$ corresponds to $\omega_{\rm RF} \tau_{4\sigma}$.

Finally, using the wakefield defined by Eq. (2.55) with the change of variables from Eq. (2.56), and the longitudinal bunch profile defined in Eq. (2.66), the induced voltage is

$$V_{\rm ind}(\phi) = -eN_b \int_{-\infty}^{\infty} \lambda(\phi') W(\phi - \phi') d\phi', \qquad (2.72)$$

where N_b is the total number of particles in the bunch, or equivalently in Fourier space

$$V_{\rm ind}(\phi) = -\frac{eN_b}{2\pi} \int_{-\infty}^{\infty} S(\omega) Z(\omega) e^{i\omega\frac{\phi}{\omega_{\rm RF}}} d\omega, \qquad (2.73)$$

where $S(\omega)$ is the bunch spectrum, defined by

$$S(\omega) = \int_{-\infty}^{\infty} \lambda(\phi) e^{i\omega\frac{\phi}{\omega_{\rm RF}}} d\phi$$
(2.74)

and $Z(\omega)$ is the beam-coupling impedance defined by Eq. (2.57).

In most analytical calculations, the induced voltage is considered as small compared to the RF field and it is assumed that, in first approximation, the particle performs an oscillatory motion given by the solution in Eq. (2.29). Using the Jacobi-Anger expansion,

$$e^{iz\cos(\omega t)} = J_0(z) + 2\sum_{n=1}^{\infty} i^n J_n(z)\cos(n\omega t),$$
(2.75)

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where J_n is the Bessel functions of the first kind and order n, the exponential function in Eq. (2.73) for small ϕ can be expanded and V_{ind} written as

$$V_{\rm ind} = -\frac{eN_b}{2\pi} \int_{-\infty}^{+\infty} S(\omega) Z(\omega) \left[J_0\left(\omega \frac{\hat{\phi}}{\omega_{\rm RF}}\right) + 2iJ_1\left(\omega \frac{\hat{\phi}}{\omega_{\rm RF}}\right) \cos(\omega_s t) + \dots \right].$$
(2.76)

Inserting Eq. (2.76) in the equation of motion (2.64)

$$\ddot{\phi} + \omega_{s0}^2 \phi \approx \frac{\omega_{s0}^2 e N_b}{V_1 \cos \phi_s} \left(Z_0 + Z_1 \phi \right), \tag{2.77}$$

where the definitions of the effective impedances Z_0 and Z_1 have been introduced as follows [39]

$$Z_0 = \int_{-\infty}^{\infty} S(\omega) Z(\omega) J_0\left(\omega \frac{\hat{\phi}}{\omega_{\rm RF}}\right) d\omega \approx_{\hat{\phi} \to 0} \int_{-\infty}^{\infty} S(\omega) \operatorname{Re} Z(\omega) d\omega, \qquad (2.78)$$

and

$$Z_{1} = \int_{-\infty}^{\infty} \frac{2i\omega_{\rm RF}}{\hat{\phi}} S(\omega) Z(\omega) J_{1}\left(\omega \frac{\hat{\phi}}{\omega_{\rm RF}}\right) d\omega \approx \int_{-\infty}^{\infty} S(\omega) {\rm Im} Z(\omega) \omega d\omega, \qquad (2.79)$$

The resistive part of the effective impedance (Z_0) contributes to the synchronous phase shift while the reactive part of the effective impedance (Z_1) contributes to the synchrotron frequency shift. These effects, time independent, are generally called the potential well distortion. The above expressions are applicable for the bunch centre only. To study the dynamics inside the whole bunch the Vlasov equation is used in the next section.

2.6 Vlasov Equation

The Liouville theorem provides the theoretical framework to study the evolution of the distribution function. For a Hamiltonian system, the distribution function is constant along any trajectory in phase space which is a solution of the equations of motion [14], i.e.

$$\frac{d\mathscr{F}}{dt} = \frac{\partial\mathscr{F}}{\partial t} + \{\mathscr{F}, H\} = 0, \qquad (2.80)$$

where {} is the Poisson bracket [40], which is expressed in the action-angle variables as

$$\{\mathcal{F}, H\} = \frac{\partial \mathcal{F}}{\partial \psi} \frac{\partial H}{\partial \mathcal{J}} - \frac{\partial \mathcal{F}}{\partial \mathcal{J}} \frac{\partial H}{\partial \psi}.$$
(2.81)

The equation (2.80) is called the Vlasov equation. For any stationary distribution $(\partial \mathcal{F}_0 / \partial t = 0)$, the Poisson bracket

$$\{\mathscr{F}_0, H\} = 0,$$
 (2.82)

and the stationary distribution \mathcal{F}_0 can be therefore expressed as a function of the Hamiltonian *H* or, equally, the action \mathcal{J} ,

$$\mathscr{F}_0 \equiv \mathscr{F}_0(\mathscr{J}). \tag{2.83}$$

The solutions of the Vlasov equation can be separated in a stationary term $\mathcal{F}_0(\mathcal{J})$ and a dynamic term $f(\mathcal{J}, \psi, t)$. With intensity effects included, the stationary term is affected by the potential well distortion only. To study the dynamic evolution of the particle distribution, we define the deviation of the perturbed distribution from its equilibrium

$$f(\mathcal{J}, \psi, t) = \mathcal{F}(\mathcal{J}, \psi, t) - \mathcal{F}_0(\mathcal{J}).$$
(2.84)

Assuming a small perturbation and keeping only linear terms in the perturbation, the Vlasov equation can be linearized and written in the form

$$\frac{\partial f(\mathcal{J}, \psi, t)}{\partial t} + \dot{\mathcal{J}} \frac{\partial \mathcal{F}_0(\mathcal{J})}{\partial \mathcal{J}} + \omega_s(\mathcal{J}) \frac{\partial f(\mathcal{J}, \psi, t)}{\partial \psi} = 0.$$
(2.85)

In the next section, the response of the beam to an external perturbation is analysed through the formalism of the beam transfer functions [35], using the linearized Vlasov equation.

2.7 Beam Transfer Function

An analytical expression of the beam response to any voltage modulation can be obtained through the linearized Vlasov's equation. The expression is not necessarily integrable analytically, but it gives indications on the leading terms for the beam instability. It is convenient for complex RF systems to consider the beam response to the perturbation separately from the wakefield generation. We use the beam transfer matrix (BTM) formalism, which expresses the amplitude and phase modulations at harmonics of the beam current, as a response to the modulation of the external voltage. Detailed calculation can be found in Ref. [35], only the results that allow interpreting beam measurements and particle simulations shown in this thesis are mentioned in what follows.

Let us assume a voltage perturbation $\tilde{V}(\phi, t)$ such that the total voltage seen by an arbitrary particle in RF phase ϕ is

$$V(\phi) = V_{\rm RF}(\phi) + \tilde{V}(\phi, t). \tag{2.86}$$

The p-th azimuthal harmonic of the beam current perturbation, $j_p(\omega)$, is related to the voltage perturbation by

$$j_p(\omega) = \sum_{k=-\infty}^{\infty} G_{pk}(\omega) \tilde{V}_k(\omega), \qquad (2.87)$$

where $G_{pk}(\omega)$ is the beam transfer matrix, defined below. It contains information about the amplitude and the phase of the beam response with respect to the voltage modulation.

To define $G_{pk}(\omega)$, we need the function $I_{mk}(\mathcal{J})$, introduced in Ref. [41], that can be written as

$$I_{mk}(\mathscr{J}) = \frac{1}{2\pi} \int_{-\pi}^{\pi} e^{i\frac{k}{\hbar}\phi(\mathscr{J},\psi) - im\psi} d\psi.$$
(2.88)

The function has the properties $I_{-mk} = I_{mk}$, $I_{m-k} = I_{mk}^*$, and for a symmetric potential well,

 $I_{mk}^* = (-1)^m I_{mk}.$

The beam transfer matrix can be written

$$G_{pk}(\omega) = -i\frac{I_0eh}{k}\sum_{m=-\infty}^{\infty} m \int_0^{\mathscr{I}_{max}} \frac{\partial\mathscr{F}_0}{\partial\mathscr{J}} \frac{I_{mk}^*(\mathscr{J})I_{mp}(\mathscr{J})}{\omega - m\omega_s(\mathscr{J}) - i\sigma} d\mathscr{J} = \sum_{m=1}^{\infty} G_{pk}^m, \qquad (2.89)$$

where the integration contour in Eq. (2.89) is chosen to satisfy the initial conditions, and $I_0 = N_b ec/(2\pi R)$ is the single bunch current in the machine. The expression (2.89) of the beam transfer function for a single bunch can be generalized for multi-bunch beams by summing over all additional bunches. The matrix elements G_{pk}^m defined by Eq. (2.89) are proportional to the dispersion integral

$$G_{pk}^{m}(\omega) \propto \int_{0}^{\mathscr{J}_{\max}} \frac{\partial \mathscr{F}_{0}}{\partial \mathscr{J}} \frac{I_{mk}^{*}(\mathscr{J}) I_{mp}(\mathscr{J}) \omega_{s}(\mathscr{J})}{(\omega - i\sigma)^{2} - m^{2} \omega_{s}^{2}(\mathscr{J})} d\mathscr{J}.$$
(2.90)

They can be split into their principal value (*P.V.*) and the residue at their poles. For positive ω and a synchrotron frequency distribution inside the bunch such that, $\omega'_s(\mathcal{J}) = 0$ only if $\partial \mathcal{F}_0 / \partial \mathcal{J} = 0$, one can write

$$G_{pk}^{m}(\omega) \propto P.V. \int_{0}^{\mathscr{J}_{max}} \frac{\partial \mathscr{F}_{0}}{\partial \mathscr{J}} \frac{I_{mk}(\mathscr{J})I_{mp}(\mathscr{J})\omega_{s}(\mathscr{J})}{\omega^{2} - m^{2}\omega_{s}^{2}(\mathscr{J})} d\mathscr{J} + i\frac{\pi}{2m^{2}} \frac{\partial \mathscr{F}_{0}}{\partial \mathscr{J}}|_{\mathscr{J}=\mathscr{J}_{0}} \frac{I_{mk}(\mathscr{J}_{0})I_{mp}(\mathscr{J}_{0})}{|\omega_{s}'(\mathscr{J}_{0})|}$$

$$(2.91)$$

From Eq. (2.91), if the synchrotron frequency derivative tends to zero in \mathcal{J}_0 , as it can be the case in a double RF system (see Section 2.3), the response of the beam to the perturbation diverges. It was concluded in Ref. [42] that the instability threshold of the bunch with nonmonotonic behaviour of the synchrotron frequency goes to zero.

However, at the resonant \mathcal{J}_0 , where $\omega = m\omega_s(\mathcal{J}_0)$, the denominator in Eq. (2.89) can be expanded around $\mathcal{J} = \mathcal{J}_0$

$$\omega - m\omega_{s}(\mathcal{J}) = m\omega_{s}'(\mathcal{J}_{0})\left(\mathcal{J} - \mathcal{J}_{0}\right) + \frac{m}{2}\omega_{s}''(\mathcal{J}_{0})\left(\mathcal{J} - \mathcal{J}_{0}\right)^{2} + \frac{m}{6}\omega_{s}'''(\mathcal{J}_{0})\left(\mathcal{J} - \mathcal{J}_{0}\right)^{3} + O\left(\mathcal{J} - \mathcal{J}_{0}\right)^{4}.$$
(2.92)

The first nonzero term in expansion (2.92) can be used to evaluate the dispersion integral.

To compute the response of p-th azimuthal harmonic of the beam current perturbation, it is convenient to transform the expression (2.87) in the time domain

$$j_p(t) = \sum_{k=-\infty}^{\infty} \int_0^t G_{pk}(t-t') \tilde{V}_k(t') dt' = \sum_{k=-\infty}^{\infty} j_p^k(t),$$
(2.93)

where the convolution theorem has been used. Detailed calculations of $G_{pk}(t)$ can be found in [35]. Assuming a periodic voltage perturbation with frequency Ω

$$\tilde{V}(t) \propto e^{-i\Omega t},\tag{2.94}$$

the modulation of the beam current has a term proportional to

$$j_{p}^{k}(t) \propto \sum_{m=-\infty}^{\infty} \int_{0}^{\mathcal{J}_{\max}} \frac{\partial \mathcal{F}_{0}}{\partial \mathcal{J}} I_{mk}^{*}(\mathcal{J}) I_{mp}(\mathcal{J}) e^{-im\omega_{s}(\mathcal{J})t} \frac{e^{i[m\omega_{s}(\mathcal{J})-\Omega]t}-1}{i[m\omega_{s}(\mathcal{J})-\Omega]} d\mathcal{J}.$$
(2.95)

Far from resonance, for $|m\omega_s(\mathcal{J}) - \Omega| t \gg 1$, one has an oscillating solution. On the contrary, approaching the resonance, this is when

$$\left|m\omega_{s}(\mathcal{J}) - \Omega\right| \ll \frac{1}{t},\tag{2.96}$$

the last term in Eq. (2.95) can be expanded to

$$\frac{e^{i[m\omega_{s}(\mathcal{J})-\Omega]t}-1}{i[m\omega_{s}(\mathcal{J})-\Omega]} \approx t.$$
(2.97)

Using the expansion (2.92) in Eq. (2.96), for $\omega'_s(\mathcal{J}_0) \neq 0$,

$$\left|\mathcal{J} - \mathcal{J}_0\right| \ll \frac{1}{\left|m\omega_s'(\mathcal{J}_0)\right|t}.$$
(2.98)

There is a band of particles near \mathcal{J}_0 for which the amplitude of the perturbation increases like t but the number of these particles decreases like 1/t [43]. Their contribution to the beam current modulation is constant, as it is the case of the normal Landau damping. However, when $\omega'_s(\mathcal{J}_0) = 0$ (see examples in Fig. 2.5), but $\omega''_s(\mathcal{J}_0) \neq 0$, the width of the resonant frequency band is given by

$$\left| \mathscr{J} - \mathscr{J}_0 \right| \ll \frac{\sqrt{2}}{\sqrt{\left| m\omega_s''(\mathscr{J}_0) \right| t}},\tag{2.99}$$

and the modulation of the beam current, in this case, grows like \sqrt{t} . Even if the instability is slow compared to the usual exponential growth, Landau damping is lost. This is the case of the bunch-lengthening mode in the double RF operation in the SPS.

It is also possible that the first and the second derivatives of the synchrotron frequency cancel (bunch-shortening mode), see also Section 2.3. In this case, the width of the resonant frequency band shrinks like

$$\left|\mathcal{J} - \mathcal{J}_0\right| \ll \left(\frac{6}{\left|m\omega_s^{\prime\prime\prime}(\mathcal{J}_0)\right|t}\right)^{1/3}.$$
(2.100)

The modulation of the beam current grows like $t^{2/3}$ in this case. The reasoning can be extended to the case where even the third derivative is zero. The instability would grow like $t^{3/4}$ if $\omega_s^{(4)} \neq 0$. It appears that more the plateau of synchrotron frequency distribution is flat (several derivatives go to zero) at resonance $\Omega = m\omega_s(\mathcal{J}_0)$, faster the instability can grow. These explanations will be used in Sections 4.2, 4.3, and 4.4 to interpret results of beam measurements and particle simulations.

2.8 Coupled-Bunch Instability Growth Rate and Threshold

In this section, the instability growth rates and their dependence on particle distribution are analysed. This information is important to determine which mode will grow predominantly as a function of the bunch length and the resonant frequency of the impedance. The impedance driving a coupled-bunch instability is narrowband, meaning that its quality factor is large, i.e. $Q \gg 1$, and its wakefield propagates over many buckets according to Eq. (2.62).

Let us consider a ring filled with *M* identical equally spaced bunches. The spectrum of the unstable beam has components at frequencies [19]

$$\omega = (n + lM)\omega_0 + m\omega_s, \tag{2.101}$$

where n = 0, 1, ..., M - 1 is the coupled bunch mode number, defining the phase shift $2\pi n/M$ between adjacent bunches, and $l = 0, \pm 1, ...,$ and m = 0, 1, ... the multipole numbers describing the inter-bunch motion. The mode m = 1 is called the dipole mode and corresponds to the oscillations of the bunch centroid position with time. The quadrupole mode m = 2 is related to an oscillation of the bunch length with time. For a coupled-bunch instability, the bunch profiles can exhibit oscillations described by a superposition of different modes defined by m and these oscillations are shifted in phase from bunch to bunch with a phase shift defined by the parameter n.

According to Eq. (2.87) and using the definition of the induced voltage in Eq. (2.73), the evolution of the perturbation is given by the following equation [41, 44–47]

$$j(k\omega_0 + \Omega) = \sum_{l'=-\infty}^{\infty} G_{kk'} Z(k'\omega_0 + \Omega) j(k'\omega_0 + \Omega), \qquad (2.102)$$

where k = n + lM and k' = n + l'M.

Instability Growth Rate

The equation (2.102) is general, but below we consider first the linear synchrotron motion in a single RF system. It means that the synchrotron frequency spread is neglected and there is no Landau damping. This assumption allows the equations to be simplified and an expression for the instability growth rate to be obtained in analytical form. In this case, the function I_{mk} defined by Eq. (2.88) can also be approximated by

$$I_{mk}(\mathcal{J}) \approx i^m J_m \left(k \sqrt{\mathcal{J}} a \right), \tag{2.103}$$

where

$$a = \sqrt{\frac{2|\eta|\omega_0^2}{hE_s\omega_{s0}\beta^2}}.$$
 (2.104)

If we assume that the different modes described by *m* are not coupled and we consider only one multipole *m*, supposing that $\Omega \ll \omega_0$, the eigenvalue problem in Eq. (2.102) can be simplified as follows

$$\frac{\Omega - m\omega_{s0}}{m\omega_{s0}} j_k = -i \frac{I_0 Meh}{\omega_{s0}} \sum_{l'=-\infty}^{\infty} g_{kk'}^m \frac{Z_{k'}}{k'} j_{k'}, \qquad (2.105)$$

where

$$g_{kk'}^{m} = \int_{0}^{\infty} \frac{d\mathscr{F}_{0}}{d\mathscr{J}} J_{m} \left(k \sqrt{\mathscr{J}} a \right) J_{m} \left(k' \sqrt{\mathscr{J}} a \right) d\mathscr{J}.$$
(2.106)

A narrowband impedance $Z(\omega)$ is considered with resonant frequency which is supposed to overlap a beam spectrum line, i.e. $\omega_r = (pM + n)\omega_0 + m\omega_s$, where p = 0, 1, ..., n = 1, 2, ..., M - 1, m = 1, 2, ..., are integers. The bandwidth of the impedance is assumed much smaller than the bunch spacing, i.e. $\Delta \omega_r \ll M\omega_0$, and its resonant frequency $\omega_r = p_r\omega_0$ is far away from beam lines $M\omega_0$. If this last condition is not fulfilled (as is often the case for the main cavity impedance), two harmonics $k_{1,2} = n + l_{1,2}M$ are simultaneously excited [47]. Otherwise, in the eigenvalue problem (2.105), only one term with $l' = l_p$ remains, where $p = n + l_pM \approx \pm p_r$

$$\frac{\Omega - m\omega_{s0}}{m\omega_{s0}} = -i\frac{I_0Meh}{\omega_{s0}}\frac{Z_p}{p}g_{pp}.$$
(2.107)

The eigenvalues give the coherent frequency shift and the eigenfunctions describe the unstable beam spectrum. Consider a binomial distribution of particles in phase space defined by function

$$\mathscr{F}_{0}(\mathscr{J}) = \frac{\mu + 1}{2\pi \mathscr{J}_{\max}} \left(1 - \frac{\mathscr{J}}{\mathscr{J}_{\max}} \right)^{\mu}, \quad \mathscr{J} \in [0, \mathscr{J}_{\max}],$$
(2.108)

with $\mu \ge 1$. Using the relation

$$p_r \sqrt{\mathscr{J}_{\max}} a \approx \pi f_r \tau,$$
 (2.109)

where τ is the bunch length, it can be shown that the growth rates for different modes *m* for constant τ can be expressed as

$$\frac{\mathrm{Im}\Omega}{\omega_{s0}} = \frac{4}{\pi^2} \frac{I_0 M \mathrm{Re}Z_p}{h V_1 \cos \phi_s} \frac{F_m^*}{f_0 \tau},$$
(2.110)

where the formfactor F_m^* is defined by

$$F_m^* = \frac{m\mu(\mu+1)}{f_r\tau} \int_0^1 x \left(1 - x^2\right)^{\mu-1} J_m^2(\pi f_r\tau x) dx$$
(2.111)

This formfactor is shown in Fig. 2.7 for different modes *m* and $\mu = 2$ as an example. It indicates which mode is expected to grow predominantly for a given binomial particle distribution and resonant frequency. When the value of $f_r \tau$ is high (\gg 1), a mixture of many modes can be excited at the same time and the instability type becomes what is called a microwave instability.

Another important quantity is the instability threshold which is given in the next paragraph.



Figure 2.7 – Formfactor F_m^* of the instability growth rates in Eq. (2.110) for $\mu = 2$ in distribution (2.108) and modes m = 1 (blue), m = 2 (orange), m = 3 (green) and m = 4 (red).

Instability Thresholds

The threshold of coupled-bunch instability due to a narrowband resonant impedance can be estimated using Eq. (2.102) (see Refs. [19, 44]) and can be written

$$R_{\rm sh} < \frac{|\eta|E}{eI_0\beta^2 M} \left(\frac{\Delta E}{E}\right)^2 \frac{\Delta\omega_s}{\omega_s} \frac{F}{f_0\tau} G(f_r\tau), \qquad (2.112)$$

where F is a formfactor defined by the particle distribution.

For a particle distribution in phase space,

$$\mathscr{F}_{0}(\mathscr{J}) = \frac{3}{4\pi \mathscr{J}_{\max}} \left(1 - \frac{\mathscr{J}^{2}}{\mathscr{J}_{\max}^{2}} \right), \tag{2.113}$$

the formfactor F = 0.3, and the function

$$G(x) = x \min \left[J_m^{-2}(\pi x) \right], \qquad (2.114)$$

is shown in Fig. 2.8. For a binomial distribution, it was demonstrated in Ref. [48] that the threshold of the coupled bunch instability can have a higher value than with the distribution (2.113).



Figure 2.8 – G function in Eq. (2.114) as a function of $x = f_r \tau$.

2.9 Conclusion

In this chapter the necessary theoretical background has been introduced. The equations of the synchrotron motion were derived and the Hamiltonian formalism of the longitudinal beam dynamics was established with the concept of RF bucket. The synchrotron frequency was analysed, especially in a double RF system. The concepts of wakefield and impedance were presented with the resonator model, used in many practical cases.

The collective motion of particles under the influence of the wakefield was also studied. The Vlasov equation, which expresses the evolution of the particle distribution in the phase space, was presented and allowed to obtain the beam transfer function. This function gives the amplitude and phase modulations at harmonics of the beam current, as a response of the modulation of the external voltage. It allowed to show the possible loss of Landau damping when the synchrotron frequency distribution exhibits a plateau inside the bunch. The beam transfer function also permitted to obtain the instability growth rate and the stability threshold for a narrowband resonator impedance.

3 SPS Intensity Limitations

The production of the HL-LHC beam with four proton batches containing 72 bunches spaced by 25 ns, with a bunch intensity of 2.3×10^{11} ppb, poses serious challenges for the SPS. With a beam loss budget of 10% which includes transverse beam scraping before extraction, removing halo particles, a beam with a bunch intensity of minimum 2.6×10^{11} ppb should be injected into the SPS. The most severe intensity limitations are in the longitudinal plane, and they can be grouped in three categories. First, the RF system of the SPS lacks RF power to compensate the beam loading. This reduces the voltage available at high beam intensity [11]. The second limitation corresponds to multi-bunch instabilities driven by the longitudinal beam-coupling impedance of the machine [7]. In addition, particles are lost from the RF bucket at capture, during flat bottom and at the start of acceleration, and losses are increasing with the beam intensity [49].

The detrimental impact of all these limitations on beam stability and quality increases with the beam intensity. Already at the nominal one $(1.15 \times 10^{11} \text{ ppb})$, the LHC proton beam is very unstable and must be stabilized using different methods during acceleration to 450 GeV/c. This chapter introduces the present intensity limitations of the SPS in the longitudinal plane, together with the mitigation measures implemented. The ways to improve the intensity limit after LS2 will be studied in Chapters 5 and 6.

In the first section, the beam loading and its effect on the beam are explained. The RF systems of the SPS are also introduced together with their limitations. The SPS longitudinal impedance model is presented in the second section of this chapter, with emphasis on the main elements impacting multi-bunch beam stability. The existence of the complete longitudinal impedance model is a key component in the understanding of the beam instabilities in the SPS. New simulation results concerning single bunch and multi-bunch instabilities are analysed in Section 3.3. The cures which allow the production of the nominal LHC beam are described in the fourth section. There, the operation in a double RF system is analysed with a focus on its effect on beam stability. An optimized voltage program during the acceleration cycle for the second RF system was obtained and implemented later in operation. The last section deals with the particle losses at injection and during the flat bottom (26 GeV/c).

3.1 RF Power Limitation and Beam Loading

The RF cavities are deliberatly tuned at (or near) a frequency where the component of the beam current is significant. Their impedance is large compared to the rest of the longitudinal impedance of the ring, and the corresponding induced voltage can increase along the beam. Additional power is needed from the RF generator to compensate this induced voltage. This is the problem of the beam loading discussed below.

The wakefield induced at the fundamental mode of a RF cavity (200 MHz in the SPS) usually decays over many bunches. When the beam enters the cavity, the induced voltage accumulates along the batch. For nominal LHC bunch intensity in the SPS, the amplitude of this voltage can become comparable to the RF voltage and drastically reduce the voltage available for the beam. The bunch length increase can lead to particle losses making impossible the injection into the 400 MHz RF system of the LHC. The effect on the beam of the induced voltage at the fundamental impedance of a RF cavity is called beam loading and it is treated in general separately with respect to the other sources of impedance.

The beam loading has a negative impact on beam quality. It causes RF amplitude and phase errors at beam injection (due to unmatched RF bucket) which may lead to an uncontrolled emittance blow-up. It may also create significant power loss, which shifts the synchronous phase along the batch. In the SPS, it complicates the double RF operation, the controlled emittance blow-up, and leads to a bunch length variation along the batch [50]. Moreover, the transient beam loading reduces the longitudinal acceptance, which limits the bunch emittance that can be captured and accelerated without particle losses, since the SPS RF bucket is full after injection (see Section 3.5). There is nevertheless one possible beneficial effect of the beam loading; it produces a modulation of the 200 MHz voltage amplitude seen by different bunches, which could decouple bunches and reduce the effect of the multi-bunch instability [51]. However, this modulation is not controlled. Simulations related to this effect are studied in Section 4.1.

The present SPS 200 MHz RF system consists of two five-section cavities and two four-section cavities, each section containing 11 cells. They are travelling-wave (TW) structures [8]. The RF generator sends a wave that propagates along with the beam. This type of cavity is used in the SPS because it provides a sufficient bandwidth to accelerate particles of different species from injection to flat top energy. A photo of one 200 MHz five-section cavity installed in the SPS tunnel before LS2 is shown in Fig. 3.1.

In addition to the main RF system, two three-section 800 MHz TW cavities, with 11 cells per section and four cells for the power couplers, are used in the SPS to improve beam stability.

The particularity of the TW structures lies in the difference between the impedance Z_g , seen by the RF generator current I_g , and the impedance Z_b , seen by the beam current I_b [8]. The RF frequency ω_{RF} increases during the cycle to follow the beam energy increase. A perfect synchronism between the revolution frequency ω_0 of the synchronous particle and the cavity



Figure 3.1 - Photo of the five-section 200 MHz TW cavity in the SPS tunnel.

central frequency ω_r , cannot be maintained throughout all the acceleration cycle. The total phase slip, during one passage of the particles, can be defined as

$$\phi_{\rm slip} = \frac{L}{v_g} (\omega_{\rm RF} - \omega_r), \tag{3.1}$$

where v_g is the group velocity and *L* is the interaction length of the cavity. For the 200 MHz TW structure, $v_g = 0.0946c$. The impedance of the cavity seen by the RF current (generator) can be written as,

$$Z_{\rm RF} = \sqrt{\frac{R_2 Z_0}{2}} \frac{\sin \frac{\phi_{\rm slip}}{2}}{\frac{\phi_{\rm slip}}{2}} L, \qquad (3.2)$$

whereas the impedance seen by the beam is

$$Z_b = -\frac{R_2}{8} \left[\left(\frac{\sin \frac{\phi_{\text{slip}}}{2}}{\frac{\phi_{\text{slip}}}{2}} \right)^2 - 2i \frac{\phi_{\text{slip}} - \sin \phi_{\text{slip}}}{\phi_{\text{slip}}^2} \right] L^2, \tag{3.3}$$

where R_2 is the series impedance of the cavity and Z_0 is the characteristic impedance of the RF chain. For the 200 MHz TW structure, $R_2 = 27.1 \text{ k}\Omega/\text{m}^2$ and $Z_0 = 50 \Omega$. As follows from Eqs. (3.2) and (3.3), the induced voltage is proportional to the cavity length square, but the RF voltage sent by the generator increases only linearly with the cavity length. This fact has two consequences regarding the beam loading. First, even though the RF voltage is already present when the beam enters the cavity, the perturbation induced by the beam grows faster along the batch than the possible correction of the RF voltage from the feedback (or feedforward) system. The evolution of both, induced and RF voltages, are sketched in Fig. 3.2. A feedback system cannot compensate perfectly the beam loading in a TW structure. Second, the total



Figure 3.2 – Schematic view of evolution, as a function of time, of the RF voltage V_{RF} (top) and the beam-loading induced voltage V_b (bottom) in the travelling-wave cavity. The filling time of the cavity L/v_g is also indicated.

impedance of shorter cavities is smaller than the impedance of longer cavities that provide the same RF voltage.

The compensation of the beam loading effect is also restricted by the limited RF power available in the SPS. When the ring is filled with equally spaced bunches, the RF power plant can deliver an average power of 0.75 MW per cavity in the present configuration. For the LHC beam, which fills only 31% of the machine, the limitation is higher. In future, the RF system can be used in pulsing mode where the voltage is zero when there is no beam in the cavity. This mode of operation allows reaching even higher peak power for the beam, which value has been measured up to 1.05 MW in the four- and five-section cavities [52], but the present low-level RF (LLRF) is not suitable for the pulsing mode.

As discussed in Chapter 5 in detail, during the LIU SPS RF upgrade, the two five-section cavities will be replaced by four three-section cavities using spare sections. Two new power plants will be used for the four-section cavities to obtain more voltage and to compensate the beam loading at higher intensity. The LLRF will be upgraded to improve the beam loading compensation (-20 dB to -26 dB) and also the pulsing mode will be possible in operation. The power plants after the RF upgrade will deliver 1.6 MW to each of the four-section cavity and 1.05 MW to each of the three-section cavity. The power plants of the 800 MHz cavities have been already upgraded during the last long shutdown. The RF parameters for the four types of SPS cavities (200 MHz and 800 MHz) are given in Table 3.1.

One figure of merit of the RF system is the maximum voltage it can provide to the beam. The input power of the cavity and the component of the beam current at the cavity frequency

Table 3.1 – RF parameters of the travelling-wave structures in the SPS. The five-, four- and three-section 200 MHz cavities are included together with the three-section 800 MHz cavities. The values of the average and peak power for the three- and four-section cavities are the one after RF upgrade, the five-section cavities will disappear after LS2. The 800 MHz RF power plant has been already upgraded during LS1.

	200 MHz			800 MHz
	5-section	4-section	3-section	3-section
RF frequency $f_{\rm RF}$ [MHz]	200.222	200.222	200.222	800.888
Interaction length L [m]	20.196	16.08	11.97	3.46
Series impedance $R_2 [k\Omega/m^2]$	27.1			647.0
Filling time L/v_g [µs]	0.712	0.568	0.422	0.330
Beam loading impedance $L^2 R_2/8 [M\Omega]$	1.381	0.879	0.485	0.968
Power (average/peak) [MW]	0.75/1.0	0.75/1.6	0.75/1.05	0.144/0.216

define the beam induced voltage. In what follows the focus will be on the 200 MHz TW structure, since for the SPS bunches the beam current component at 800 MHz is much smaller.

Assuming no phase slippage between the travelling wave and the bunch, i.e. $\phi_{slip} = 0$ (RF voltage and beam loading are maximum), the power required to provide a voltage V_n in one travelling wave structure with *n* sections can be written [11] as

$$P_n = \frac{V_n^2}{R_2 L_n^2} + \frac{R_2 L_n^2}{64} I_{\rm RF}^2 + V_n I_{\rm RF} \frac{\sin \phi_s}{4},\tag{3.4}$$

where $L_n = L_0(11n - 1)$ is the interaction length of the n-section cavity, L_0 is the length of one cell (for the 200 MHz TW structure $L_0 = 0.374$ m) and $I_{RF} = 2FN_be/T_{bb}$ is the Fourrier harmonic with $F = S(\omega_{RF})$, where $S(\omega)$ is the normalized bunch spectrum given by Eq. (2.74). For short bunches, $F \approx 1$. The first term in Eq. (3.4) is the power needed to generate the cavity voltage V_n , seen by the beam. The second term corresponds to the beam loading and the last term is the additional power needed to accelerate the beam. This last term is zero at the flat top and the flat bottom energies ($\phi_s = \pi$).

Using the parameters from Tab. 3.1, the total 200 MHz voltage available at flat top (short bunches) in the presence of beam loading is shown in Fig. 3.3 as a function of the bunch intensity for the present RF configuration and the one after LIU RF upgrade. The voltage limitation was close to 7 MV in nominal operation $(1.15 \times 10^{11} \text{ ppb})$ and after RF upgrade it will be raised to 10 MV for HL-LHC intensity ($2.3 \times 10^{11} \text{ ppb}$). However, the first power limitation appears already during acceleration.

The operational voltage program for the LHC beam in the SPS is designed to keep the momentum filling factor q_p (ratio of the $2\sigma_p$ bunch height to the bucket height in momentum) constant during the first part of the acceleration to mitigate particle losses. The momentum



Figure 3.3 – Maximum 200 MHz RF voltage available for the LHC-type beam with 25 ns spacing at flat top (short bunches) as a function of the bunch intensity with the present (blue) and the upgraded (orange) RF power plant. The dashed vertical and horizontal lines correspond to the present limitation of 7 MV with nominal LHC bunch intensity and the future limitation of 10 MV with HL-LHC bunch intensity. If F < 1 (as it is the case in the SPS), 10 MV is available for a bunch intensity of 2.4×10^{11} ppb.

filling factor is defined by the following expression

$$q_p = \frac{\Delta E_{\text{bunch}}}{\beta^2 \Delta E_{\text{bucket}}},\tag{3.5}$$

where ΔE_{bunch} is the maximum energy deviation from E_s within the bunch and ΔE_{bucket} is the bucket height in energy. From the middle of the ramp the voltage is kept at a high value, since a higher beam stability was obtained in this case [53] and also for an eventual controlled emittance blow-up. Then, the voltage is increased to 7 MV at flat top. Due to the last term in the r.h.s. of Eq. (3.4), the power demand increases sharply when the acceleration starts. The operational voltage programs for the two RF systems are shown in Fig. 3.4, together with the RF power required during the cycle in the four-section and five-section cavities for a nominal bunch intensity ($N_b = 1.15 \times 10^{11}$ ppb). For constant q_p , the maximum power is needed at the start of the acceleration and on the flat top. The voltage program during the cycle can be computed for given bunch emittance and bucket filling factor. Examples are shown in Fig. 3.5 (a) for a nominal injected emittance of 0.35 eVs and different filling factors. By iterating on emittances and filling factors, it is possible to obtain the maximum power as a function of the bunch emittance for a given filling factor, as shown in Fig. 3.5 (b) for $q_p = 0.80$ and a nominal LHC bunch intensity ($N_b = 1.15 \times 10^{11}$ ppb). The filling factor is kept low because of the full



Figure 3.4 – The operational voltage programs during the SPS acceleration cycle. The voltage of the second RF system (800 MHz) is fixed at 10% of the voltage at 200 MHz. The power in four-section and five-section cavity is also shown for a bunch intensity of 1.15×10^{11} ppb.



Figure 3.5 – (a) Voltage programs for the injected bunch emittance of 0.35 eVs and different momentum filling factors q_p during the cycle. (b) The maximum power per cavity required during the cycle for nominal bunch intensity (1.15×10^{11} ppb) as a function of the emittance for a filling factor $q_p = 0.80$. The horizontal dashed line corresponds to the maximum average power available of 0.75 MW.

bucket at the start of the ramp in the SPS. The value $q_p = 0.8$ is, in general, used to reduce particle loss at the bucket edges. If the RF power is limited to 750 kW per cavity, we obtain the maximum emittance that can be accelerated as a function of the filling factor. Results for the

present 200 MHz RF system are shown in Fig. 3.6.



Figure 3.6 – Maximum emittance that can be accelerated in the SPS for a power of 0.75 MW per cavity as a function of the constant filling factor q_p during the cycle. The bunch intensity is nominal $(1.15 \times 10^{11} \text{ ppb})$.

3.2 Longitudinal Impedance Model of the SPS

A survey of main impedance sources in the SPS ring was performed over many years [3, 8, 54–58]. They have been characterized using calculations, RF measurements and electromagnetic simulations. The longitudinal impedance model of the SPS contains more than 200 resonant peaks, both broad and narrowband, in the range from 50 MHz (kickers, see below) to 2.5 GHz. The present longitudinal impedance model is shown in Fig. 3.7. High narrowband peaks could be responsible for the coupled-bunch instability (see Section 3.3). The real and imaginary parts of this impedance model will be given at the end of this section. Some impedance above 2.5 GHz is also included in the model, but not shown here. The cut-off frequency for most of the various SPS beam pipes is below this frequency. The cut-off frequency is the frequency above which a perturbation would not resonate in the structure but travel through the beam pipe. The amplitude of the typical stationary bunch spectrum in the SPS is also negligible above 2.5 GHz. The main contributions of this rather complicated impedance model are described below.

200 MHz Travelling-Wave Structures

The main RF system of the SPS (200 MHz) is a leading contributor to the SPS impedance budget. The total impedance of the four 200 MHz cavities (see also Section 3.1) is shown



Figure 3.7 – Present longitudinal impedance model of the SPS (before LIU upgrades). The modulus of the impedance is shown.

in Fig. 3.8, where both the accelerating and the Higher Order Mode (HOM) bands of the fiveand four-section cavities contribute significantly. The fundamental impedance in the 200 MHz passband (max(Z_b) ~4.5 MΩ) is reduced in operation with the LHC beam by -20 dB (factor 10) by the one-turn-delay feedback (OTFB) and feedforward (FF) systems (see Section 3.4.1) [16]. Only the reduced value is shown. There are four HOM bands: the 630 MHz band which is already heavily damped by RF couplers [59], the 914 MHz band which is also affected by RF couplers, the 1.13 GHz band and the 1.50 GHz band. A photo of the RF coupler installed for damping of the 630 MHz HOM is shown in Fig. 3.9 together with a Computer Assisted Drawing



Figure 3.8 – Longitudinal impedance of the 200 MHz RF system in the SPS (orange) together with the full model (blue). The accelerating band and the HOMs bands are shown for the two five-section cavities and the two four-section cavities.



(a) Photo of the 630 MHz coupler

(b) CAD drawing of the coupler installed

Figure 3.9 – RF couplers used in the SPS to damp the HOM at 630 MHz. The photo of the present coupler (a) and a CAD of the coupler in a cavity cell (b) [60].

(CAD) of the coupler inside the cavity cell. Without this HOM coupler, the 630 MHz impedance would be larger by a factor of about 50. It will be shown in Chapter 5 that the 630 MHz HOM has a significant impact on beam stability and further impedance reduction is needed for HL-LHC beam stability.

Vacuum Equipment

The vacuum flanges are another important source of the SPS impedance. They are included in the model together with other vacuum equipment like the sector valves [61] and the unshielded pumping ports [55]. Their impedance is at high frequencies, above 1 GHz. Two main types of vacuum flanges can be distinguished by the quadrupole magnet type and the corresponding shape of the beam pipe at their location. The flanges of the QF-type are used when the beam pipe is elliptical, as shown in Fig. 3.10 (a) and the flanges of the QD-type are used when the beam pipe is cylindrical, Fig. 3.10 (b). Due to the difference in size and shape, their resonant frequencies are also different. Depending on the location, the flanges have bellows of different



Figure 3.10 – Computer Assisted Drawing (CAD) of the QF-type (a) and the QD-type (b) of vacuum flanges in the SPS [62].

lengths.

Most of the pumping ports (~ 800) have been shielded in the previous impedance reduction campaign [63], their impedance can be neglected and those remaining unshielded (25) are included in the model [64].

The frequency range of flanges in the model also includes other contributions like the sector valves, which are used to close parts of the machine in case of vacuum leaks or interventions, and some other vacuum chambers with different shapes. The impedance of the three flanges categories (QD, QF, others) is shown in Fig. 3.11.

Kickers

The SPS impedance model contains seven fast extraction kickers, named MKE, shielded by serigraphy [65]. It also contains 16 kicker magnets for proton injection (MKP) which remain unshielded and seven other kickers to extract the beam to the dump (MKD) and to measure the transverse tune and the aperture (MKQ). Their impedance is of a broadband type centred around $f_r \sim 1$ GHz with some narrowband peaks at low frequency of ~ 50 MHz, which can be a source for beam induced heating of the kicker magnets. The total impedance of the different kickers is shown in Fig. 3.12. The impedances of the MKE and MKP kickers have a significant impact on beam stability. The MKE kickers have been already shielded and serigraphed, and at the moment no more impedance reduction is planned. The impact of impedance reduction of the MKP on beam stability is analysed in Chapter 5.



Figure 3.11 – Impedance of the vacuum equipment in the SPS including the QD flanges (green), the QF flanges (red) and other vacuum hardware together, including the vacuum valves (orange).



Figure 3.12 – Impedance of the kickers in the SPS. The contributions from the MKD kickers (orange), the MKE (green), the MKP (red) and the tune kickers (purple) are shown [54, 66].

Present Longitudinal Impedance Model of the SPS

The longitudinal impedance model of the SPS also contains the main and HOM impedances of two 800 MHz cavities which can be described by an expression similar to Eq. (3.3) with different parameters R_2 and L given in Tab. 3.1 [54].

Many smaller contributions are also included, from beam instrumentation devices, to the resistive wall impedance and the space charge [28] (non-negligible at 26 GeV/c). The real part of the SPS impedance model is shown in Fig. 3.13 (a) with the main contributions indicated and the imaginary part of the impedance, normalized by $n = f_r/f_0$, is shown in Fig. 3.13 (b). The parameters of the different resonators are listed in Appendix A.

It is important to mention that the SPS impedance model was evolving over the years. The model shown here is the latest version at the beginning of 2019, and, if not specified, the simulations presented in this thesis use this model. Due to the complexity of the model, it was benchmarked by reproducing beam measurements in simulations [6,22,39,67]. It was found [67] that some longitudinal impedance might still be missing in the model. Later, it was also revealed that the impedance of the MKE kickers has been underestimated in the past [66]. Remarkably, the additional contribution of the MKE kickers has a very similar characteristics to the aforementioned missing impedance.



Figure 3.13 - Present longitudinal impedance model of the SPS (before LIU upgrades). The real part of the impedance (a) and the ImZ/n (b) are shown. The major contributions are indicated in different colours, and the total SPS impedance is shown in blue.

3.3 Longitudinal Beam Instabilities in the SPS

The previous successful SPS impedance reduction campaign, in the years 1999–2000, eliminated the microwave instability observed for a single bunch on the SPS flat bottom. Presently the single bunch becomes unstable in the last part of the ramp for bunch intensities above nominal $(1.15 \times 10^{11} \text{ ppb})$.

The longitudinal impedance was also probed using beam measurements. Results for the synchrotron frequency shift and the stability threshold of a single bunch in the single and

the double RF systems during the acceleration ramp were presented in [53, 68]. Some discrepancies were observed between beam measurements and simulation results that could have been explained by missing impedance in the longitudinal impedance model [68] used at that time or perturbations in the particle distribution in phase space, not well reproduced in simulations. Indeed, it was found later that the impedance of the MKE kicker has been underestimated [66]. An inaccuracy has also been found in the QD flange impedance of the vertical beam position monitor (BPV).

New simulations of single-bunch instabilities during acceleration with the latest longitudinal impedance model (Fig. 3.7) give better agreement with measurements. The same procedure as described in Ref. [39] was followed to simulate the beam stability with the latest longitudinal impedance model. The measured bunch profiles after filamentation at flat bottom were used to generate the particle distribution in the phase space matched to the RF bucket including intensity effects. The momentum and voltage program during the cycle were the ones used in operation for a single RF system (200 MHz) at that time and they are shown in Fig. 3.14 (left). The voltage at the flat bottom energy was smaller than the 4.5 MV used in the LHC cycle since the bunch emittance was smaller (~0.25 eVs) in these measurements. The momentum cycle is also shorter than the LHC cycle in Fig. 1.2 to increase the number of measurements that could be achieved. In simulations, the bunch was tracked during acceleration (from 26 GeV/c to 450 GeV/c). The bunch length at the end of the cycle measured and simulated with the longitudinal SPS impedance model, without the corrections mentioned above, is plotted in Fig. 3.14 (right). The bunch lengthening in simulations is close to the one measured. However, the instability onsets in simulations at smaller bunch intensity than in beam measurements. With the updated impedance model, the bunch lengthening due to potential well distortion is also well reproduced, as shown in Fig. 3.15, however, the bunch intensity at which the instability onset agrees well this time.

The single-bunch instability can be understood with the latest longitudinal impedance model



Figure 3.14 – Left: momentum (black) and RF voltage program (blue) from Ref. [39] used in simulations and beam measurements with a single bunch. Right: length of a single bunch at the end of the cycle at 450 GeV/c in measurements (blue dots) and in simulations (red dots) from Ref. [39] as a function of the bunch intensity after injection.



Figure 3.15 – Length of a single bunch at the end of the cycle at 450 GeV/c as a function of the bunch intensity after injection. The measurements [39] (blue dots) are compared with macroparticle simulations (orange dots) using the latest longitudinal impedance model. A single RF system (200 MHz) is used, with the voltage program from Ref. [39], with a voltage of 7 MV at flat top (450 GeV/c).

and the study of multi-bunch instability with this model is justified. Notice that even the singlebunch stability threshold ($\sim 1.7 \times 10^{11}$ ppb) is well below the HL-LHC intensity (2.4×10^{11} ppb). The threshold will be improved thanks to the SPS impedance reduction during LS2. Due to the SPS RF upgrade a larger voltage will also be available at flat top, allowing for larger emittance.

In present operation, beam intensity is limited by the beam loading and multi-bunch instabilities during acceleration. In a single RF system, the intensity limit at flat top, even for 12 LHC bunches, is approximately $(3 - 4) \times 10^{10}$ ppb [10], well below nominal value. This is a severe limitation, since the maximum bunch length allowed for injection into the 400 MHz bucket of the LHC is fixed at 1.9 ns and the extraction is controlled by the Beam Quality Monitor (BQM) [69] with an average value along the batch of 1.65 ns. This means that the longitudinal bunch emittance cannot be increased arbitrarily by uncontrolled longitudinal emittance blowup (instability) or by controlled longitudinal emittance blow-up if the RF voltage available at flat top is not increased significantly. The increase of the bunch spacing, to reduce the amplitude of the wakefield from bunch to bunch, is also not possible, since the luminosity of the HL-LHC beam would decrease. A bunch spacing of 50 ns has been used in the past for the LHC beam, but a high bunch intensity creates problems of high pile-up for the LHC experiments. This bunch spacing is not considered in this thesis. The number of bunches in the batch cannot be significantly reduced without increasing the LHC filling time, and potential penalization of the LHC luminosity. The injection of an unstable beam into the LHC could damage the machine due to particle losses. The instability must be mitigated, keeping the beam parameters required to reach the goals of the HL-LHC project.

An example of instability developing during the SPS cycle is shown in Fig. 3.16. A batch of 48 bunches was injected with nominal emittance (0.35 eVs) and an average intensity per bunch $N_b = 1.75 \times 10^{11}$ ppb. The double RF system was used with the LHC RF voltage program (Fig. 3.4) with a voltage ratio between the two RF system of 0.15. The evolution of the average bunch length with the minimum and maximum deviations along the batch is shown in Fig. 3.16 (a). The average bunch length after filamentation is about 2.75 ns and it is reducing during acceleration due to the increase of voltage and energy. However at flat top, see Fig. 3.16 (b), the average bunch length and the deviation from the average increase, which are signs of the onset of the instability. Notice that the sample rate of the measurements was increased at flat top.

The evolution of the bunch profiles for a sample of four representative bunches in the batch is shown in Fig. 3.17. The range of the x-axis covers a full RF bucket and the centre of each plot corresponds to the bucket centre. The stable position of the bunch is moving to the left along the batch (synchronous phase shift). The instability is growing along the batch, bunches at the head of the batch are stable and bunches at the tail are very unstable. In this example, bunches in the batch centre are also stable. There is a difference between the head and the tail of the batch, since the beam covers only a small fraction of the SPS ring. The tail of the batch is not affecting the head through the beam gap.



Figure 3.16 – Average bunch length (blue) measured during acceleration cycle for a batch of 48 bunches with a bunch intensity of 1.75×10^{11} ppb (a). The minimum and maximum deviation from the average value are shown with red dots. The sample rate of the measurements was increased at flat top (b). A running average has also been applied to the minimum/maximum bunch length. The Q20 optic was used in a double RF with the nominal V_{200} cycle and $V_{800}/V_{200} = 0.15$. The controlled emittance blow-up was activated during the ramp (~ 15 s).


3.3. Longitudinal Beam Instabilities in the SPS

Figure $3.17 - \text{Evolution of the bunch profile at the end of the cycle (450 GeV/c) for bunch 2 (upper left), bunch 11 (upper right), bunch 37 (lower left) and bunch 47 (lower right). The colours represent the amplitude of the bunch line density in arbitrary units. The range of the x-axis covers the full RF bucket and the centre of the plot corresponds to the bucket centre.$

The unstable bunches exhibit dipole and quadrupole oscillations (see Section 2.8). It means that bunch position and bunch length oscillate with time. To obtain the frequency of these oscillations, a Fourier transform was applied to the measured signals in time domain for both bunch position (dipole mode) and bunch length (quadrupole mode). The results are shown in Fig. 3.18. The observed instability is a superposition of different modes. For the unstable bunches, the oscillation amplitude is growing for both dipole and quadrupole modes. The frequency of the dipole oscillations is close to f_{s0} (in this double RF system $f_{s0} \sim 330$ Hz) and the frequency of the quadrupole oscillations is about $2 \times f_{s0}$, as expected for coherent instabilities, see Section 2.8. The spread of the frequency lines in Fig. 3.18 can be explained by the synchrotron frequency spread within the bunch and the finite window length (0.33 s) used to perform the Fourier transform.

The parameters of potential elements in the longitudinal impedance model (see Section 3.2) that could cause these oscillations can be estimated. The instability growth rate is proportional to a formfactor, different for every mode (Eq. (2.111)). For a given particle distribution in phase space, the formfactor is a function of the resonant frequency of the impedance and the bunch length. Values of the formfactor for the first four modes are shown in Fig. 3.19 for a binomial particle distribution (defined by Eq. (2.108)) with $\mu = 1.5$, similar to the one measured at SPS flat top (450 GeV/c). For the average bunch length at flat top of 1.65 ns, the



Figure 3.18 – Fourier transform of the bunch position (dipole, blue) and the bunch length (quadrupole, orange) for bunches from Fig. 3.17. A window of the last 0.33 s of the flat top is used to perform the Fourier transform.



Figure 3.19 – The formfactors of the instability growth rate as a function of $f_r \tau$, the resonant frequency multiplied by the bunch length. A binomial particle distribution with $\mu = 1.5$ is used, similar to the one measured at flat top.

HOM at 630 MHz of the 200 MHz cavities would trigger mainly dipole and quadrupole modes. This indicates that the coupled-bunch instability could indeed be due to the 630 MHz HOM. However, the complexity of the SPS instabilities lies in the interplay between different parts of the impedance model (broadband impedance and narrowband peaks). This is why the figure of merit of beam stability in the longitudinal plane is the intensity threshold, also called

stability (or instability) threshold. The way of simulating the intensity thresholds is explained in what follows.

3.3.1 Simulations of Stability Threshold at SPS Flat Top

The intensity thresholds were simulated with the code BLonD [20]. The motivation for these studies was the evaluation of the possible impedance reduction in the SPS for improving the HL-LHC beam stability. The onset of instability is sensitive to the particle distribution in phase space. It can also be strongly affected by the low-level RF control loops, which are difficult to model in simulations. When possible, the low-level RF was deactivated in measurements to ease their reproduction in simulations. Stable bunches after acceleration are assumed to be matched at flat top to the RF bucket with intensity effects.

In simulations, a batch of 72 bunches spaced by 25 ns is generated with particle distributions which are described by the binomial function defined by Eq. (2.108). Each particle distribution is generated randomly for every bunch using different seeds. In agreement with measurements, the parameter μ is chosen to be 1.5. The bunch length is computed, in simulations and measurements, through the FWHM of the bunch profile, τ_{FWHM} , rescaled to 4σ assuming a Gaussian distribution, $\tau_{4\sigma}$, as defined in Eq. (2.71). In the SPS, this relation is often used even for non-Gaussian bunches. The bunch emittance and the bunch intensity were scanned to obtain a stability map. The simulated time at flat top was 2.3 seconds (compared to the 500 ms in the SPS operation) to observe slowly growing instabilities. However, in relevant intensity range, close to 2.5×10^{11} ppb, the multi-bunch instabilities are violent and appear before 500 ms.

Let us define in the batch at every revolution period $\tau_{\max}(t)$, the biggest bunch length, $\tau_{\min}(t)$, the smallest bunch length, and $\bar{\tau}(t)$, the average bunch length. The amplitude of the bunch length oscillations during the cycle, normalized by the average bunch length along the batch, $\Delta \tau$, is defined by the following expression,

$$\Delta \tau = \frac{\tau_{\max}(t) - \tau_{\min}(t)}{\bar{\tau}(t)},\tag{3.6}$$

and its maximum value is used as a criterion to separate stable beams from unstable. When $\Delta \tau$ exceeds 0.07 during the cycle, the beam is considered as unstable. Other values have also been tested in simulations at flat top and the threshold depends weakly on the criteria for values between 0.05 and 0.12.

An example of a stability map obtained at flat top for a single RF system is shown in Fig. 3.20. The line which separates the stable bunch from the unstable was fit using a function of the third power of the bunch length, since the minimum coupled-bunch instability threshold scales like ϵ^2/τ , see Eq. (2.112).

Simulations of the LHC beam with the operational momentum and voltage cycle in a double



Figure 3.20 – Example of the simulated stability map. A beam of 72 bunches is matched to the RF bucket at flat top in a single 200 MHz RF system (7 MV). The colours show the relative bunch length spread from Eq. (3.6) as a function of the bunch length and the bunch intensity. The stability threshold is defined as the line separating stable and unstable beams.

RF system were carried out to study the effect of different sources on beam stability. In Fig. 3.21, the case with the longitudinal SPS impedance model is shown in a double RF (DRF) and single RF system (SRF). It was found in simulations for 72 bunches at flat top energy that the vacuum flanges impedance has a similar impact on the stability threshold as improving the HOM damping, as shown in Fig. 3.21. A longitudinal impedance model, where the HOM at 630 MHz is removed, gives a stability threshold comparable to a model without the QF flanges. When both, HOM at 630 MHz and QF flanges are removed from the impedance model, the stability threshold increases significantly. This is the upgrade scenario adopted by LIU [3]. The effect of high-frequency impedance of vacuum flanges was observed, in the past, by debunching a bunch with RF off [70, 71]. A significant component, at 1.4 GHz, was measured in the unstable bunch spectrum.

In the next section, the mitigation measures proposed for implementation in the SPS, which will allow the production of the LHC beam, are discussed in more detail.

3.4 Mitigation Measures

3.4.1 Beam Loading Compensation

To compensate the beam loading, the 200 MHz LLRF has two dedicated systems [16]. The first one is the one-turn-delay feedback (OTFB). As input, the total voltage seen by the beam is measured, this signal is filtered, and reinjected into the cavity with the proper phase. The



Figure 3.21 – Intensity threshold at flat top for 72 bunches in a double RF system with $V_{200} =$ 7 MV and $V_{800}/V_{200} =$ 0.1. The full impedance model is used (solid blue) and compared with a case where the 630 MHz HOM (red) or the vacuum flanges of the QF type (orange) are removed. The case, where both QF flanges and the HOM are removed, is also shown (green). The dot for the reference measurement [7] is included. The error bars represent the maximum and minimum bunch length around its average value. This spread of the bunch length is omitted in simulations. The single RF case with the full impedance model is also shown (dashed blue).

impedance seen by the beam is significantly reduced (~ -15 dB) but in the vicinity of the revolution frequency harmonics and the impedance reduction for the quadrupole modes at $f_{\rm RF} + nf_0 + 2f_s$ is less efficient. In this thesis, when the feedback is mentioned it always refers to the OTFB.

The second LLRF system which compensates the beam loading is the feedforward (FF) system. It reduces the transient beam loading induced by the entrance of the head of the beam in the 200 MHz TW structure. The beam current is measured upstream to the cavity and the feedforward system extrapolates (in the same revolution period) the necessary increase of the input voltage. Due to the feedforward, the total voltage seen by the beam varies from bunch to bunch in the head and the tail of the batch, which gives a particular stable phase error presented below in Fig. 3.23.

In the SPS, the one-turn-delay feedback and the feedforward systems are usually used in pairs. Together they reduce the impedance seen by the beam at revolution frequency harmonics by -20 dB [16].

The residual beam loading in simulations was modelled in two ways. In the first model, the impedance reduction is assumed to be -20 dB for the full 200 MHz impedance. This model is used for the simulations at flat top, but it cannot account for the transient beam loading

(significant for beam loss studies) and reproduce the bunch-by-bunch synchronous phase shift along the batch. This last effect is also important to explain the non-uniform controlled emittance blow-up during the ramp in the double RF system observed along the batch [50]. In the second feedback model, which can be used to better reproduce the beam parameters along the batch, the transfer functions of the electronic systems are applied [16]. The impedance seen by the beam after the OTFB and the FF corrections, Z_{bc} , can be written

$$Z_{bc} = \frac{Z_b - H_{FF} Z_{\rm RF}^2}{1 + H_{FB} Z_{\rm RF}^2},$$
(3.7)

where H_{FF} and H_{FB} are the FF and the OTFB gains, respectively, with Z_{RF} and Z_b defined in Eq. (3.2) and 3.3, respectively. Their values have been calibrated from beam measurements [72],

$$H_{FF} = \frac{0.02}{L^2 R_2}, \qquad H_{FB} = \frac{20.00}{Z_0 L^2 R_2}.$$
 (3.8)

The resulting beam impedances obtained in the two models are compared in Fig. 3.22 (a) to the impedance without OTFB nor FF system (Z_b). Both the OTFB and the FF have limited bandwidth (~3 MHz), which is the reason why the impedance in the second model has side-lobes with higher impedance. These side-lobes produce a modulation of the induced voltage along the batch. The envelope of the induced voltage, for the three cases, is shown in Fig. 3.22 (b) for a batch of 72 bunches with an average bunch length of 1.65 ns and bunch intensity (average) $N_b = 1.15 \times 10^{11}$ ppb at flat top. The voltage-amplitude modulation along the batch modifies the synchronous phase along the batch in all three cases. For a reduction of -20 dB or no reduction, the bunches are shifted towards the head of the beam and the shift increases along the batch. For a nominal LHC batch at flat top, the bunch position shift saturates when



Figure 3.22 – Seen by the beam total impedance of the two five-section cavities and the two four-section cavities (a) and envelope of the voltage induced by a nominal batch of 72 bunches $(N_b = 1.15 \times 10^{11} \text{ ppb}, \bar{\tau} = 1.65 \text{ ns})$ (b). The case without compensation (blue) is compared to the model with constant impedance reduction (orange) and the model using Eq. (3.8) (green).

the induced voltage reaches its maximum (after ~ 24 bunches). The maximum synchronous phase shift is about 680 ps in the case without the OTFB and the FF systems and 70 ps for the model 1. Results of simulations for 72 bunches at flat top are shown in Fig. 3.23. In the second model, where Eq. (3.8) is used, the synchronous phase shift in the batch centre is smaller than in the -20 dB case, but the head and the tail of the batch have a phase shift of ± 100 ps compared to the batch centre. This is the effect of the FF system.

The effect of the beam loading on the beam can also depend on the transition energy of the machine. The SPS uses a variety of magnetic multipoles, e.g., dipoles to fix the beam trajectory, quadrupoles to focus the beam. In the transverse plane, the settings in some of these magnets define the optics of the machine. The SPS optics is named after its corresponding transverse integer tune. The three optics available in the SPS are Q20, Q22 and Q26 and their characteristics are listed in the Table 3.2. Each optics gives rise to a different transition energy. Well above transition for a given beam energy, when the transition energy increases, the slip factor η decreases. At flat bottom, for given longitudinal bunch emittance, a lower transition energy requires a higher matched voltage, which is more favourable regarding the beam loading since the relative effect of the induced voltage with respect to the RF voltage is weaker. The matched voltage (including intensity effects) is desirable to be used since the longitudinal bunch emittance should be as small as possible at the end of flat bottom to reduce the beam losses during acceleration.



Figure 3.23 – Synchronous phase shift along the nominal batch of 72 bunches at flat top in a single RF system with 7 MV at 200 MHz and a bunch intensity $N_b = 1.15 \times 10^{11}$ ppb with an average bunch length $\tau = 1.65$ ns . The cases without beam loading compensation (blue), the first model with -20 dB reduction (orange) and the second model with the OTFB and FF described by Eq. (3.7) (green). The horizontal line (black) shows the position without beam loading (200 MHz bucket centre).

	Machine optics		
	Q20	Q22	Q26
Transition gamma $\gamma_{\rm tr}$	18.0	20.0	22.8
Slip factor η (flat top/flat bottom) [10 ⁻³]	1.8/3.1	1.2/2.5	0.6/1.9
Voltage for 0.35 eVs (FB) [MV]	4.5	3.0	2.0

Table 3.2 – Parameters of the three optics available in the SPS.

At flat bottom the RF system is not limited by its power and, for a given injected emittance, a higher slip factor also increases the stability threshold. On the contrary, in the first part of the ramp, the use of a higher transition energy would alleviate the power limitation since the required voltage is smaller. At flat top, the higher stability for larger η is compensated by the higher required voltage. Indeed, the bunch length of 1.65 ns is fixed for injection into the 400 MHz RF system of the LHC and, for a 200 MHz voltage limited to 7 MV, the emittance increases linearly with the transition energy according to Eq. (2.70) and far from transition $\eta \sim \gamma_{\rm tr}^{-2}$, according to Eq. (2.10). Therefore the term $\eta \epsilon^2$ in the calculation of the stability threshold (Eq. (2.112)) is constant. The stability thresholds for the three optics are similar as it is shown in Fig. 3.24. This figure was obtained with the longitudinal impedance model of the SPS available in April 2017, when the choice of the optics after LIU upgrades was still in discussion but this fact does not change the conclusion that all the three thresholds are similar. Finally, the Q20 optic is a better choice regarding beam stability since it was also demonstrated in simulations that the beam stability can be an issue at flat bottom in the Q22 optic.



Figure 3.24 – Intensity threshold of 72 bunches spaced by 25 ns at SPS flat top in a double RF system with $V_{200} = 7 MV$ and $V_{800}/V_{200} = 0.1$ for the three different optics available in the SPS. The longitudinal SPS impedance model was the version used in April 2017 when the choice of the optics to be used after LIU upgrades was still under discussion.

In the next section, the instability mitigation in the SPS double RF system is analysed.

3.4.2 Double RF Operation in the SPS

For nominal LHC intensity $(1.15 \times 10^{11} \text{ ppb})$, the multi-bunch instabilities in the SPS are cured by the second RF system operating at 800 MHz. This system provides more efficient Landau damping of beam instabilities [9] by increasing the synchrotron frequency spread within the bunch. The 800 MHz RF system is used throughout the entire cycle. The total voltage $V(\phi)$ seen by the particle at position in phase ϕ (measured in RF radians at 200 MHz) is defined by Eq. (2.45) and can be written as

$$V(\phi) = eV_{200} \left[\sin \phi + r \sin(n\phi + \phi_{800}) \right], \tag{3.9}$$

where $r = V_{800}/V_{200}$, V_{200} and V_{800} are the amplitude of the 200 MHz and 800 MHz voltage, respectively, n = 4, and ϕ_{800} is the relative phase between the two RF systems. Even for small r, the relative phase ϕ_{800} can significantly modify the shape of the synchrotron frequency distribution. In a double RF system, the synchrotron frequency in the bunch centre, $f_s(0)$, is changed from the value of the linear synchrotron frequency $f_{s0}(0)$ in a single RF system from Eq. (2.26) according to the expression

$$f_s^2(0) = \frac{f_{s0}^2(0)}{\cos\phi_{s0}} [\cos\phi_s + 4r\cos(4\phi_s + \phi_{800})].$$
(3.10)

As shown in Section 2.3, at flat bottom or flat top energy, the values of ϕ_{800} maximizing the effect of the 800 MHz RF system on the synchrotron frequency in the bunch centre are 0 (bunch-lengthening mode) and π (bunch-shortening mode) with $\phi_s = \phi_{s0}$. The effect on



Figure 3.25 – Bunch profile (solid blue line) and potential well (dashed black line) in a double RF system for n = 2 in bunch-shortening mode (left) and bunch lengthening mode (right). The voltage ratio between the two RF systems is r = 0.25.

the bunch profile of these two modes of operation in the SPS is illustrated in Fig. 3.25 and the synchrotron frequency distributions with r = 0.1 are compared to the case with a single RF system in Fig. 3.26 (left). The voltage ratio in SPS operation so far was r = 0.1 because it provides stability for the nominal LHC beam during the whole cycle.

The two modes of operating the 800 MHz system increase the synchrotron frequency spread and stabilize the beam according to Eq. (2.112). The bunch-lengthening mode (BLM) is, generally, attractive because it also reduces the peak bunch current and increases the bucket area. However, it drives the SPS beam unstable because the zero of the synchrotron frequency derivative (see Section 2.8) is closer to the bunch centre. Moreover, the synchrotron frequency spread Δf_s , defined as

$$\Delta f_s(\epsilon) = \frac{f_s(0) - f_s(\epsilon)}{f_{s0}(0)},\tag{3.11}$$

also strongly depends on the error in ϕ_{800} in BLM, which is difficult to control in operation. For the same phase shift, due to beam loading for example, the synchrotron frequency spread is more reduced in BLM than in BSM. The case is illustrated in Fig. 3.26 (right) for a phase shift $\Delta\phi_{800} = 28^{\circ}$ (100 ps), as observed in the SPS between the head and the tail of a batch of 72 bunches with nominal intensity when the OTFB and the FF are activated (Fig. 3.23). The spread Δf_s is also shown in Fig. 3.27 at SPS flat bottom (left) and flat top (right) as a function of the relative phase ϕ_{800} . In the SPS, the bunch-shortening mode is used during the whole cycle and this is assumed below.

During acceleration, in operation and simulations, the relative phase between the two RF systems is approximated by the following expression

$$\phi_{800} = \pi - 4\phi_{s0},\tag{3.12}$$



Figure 3.26 – Left: normalized synchrotron frequency distribution in a double RF system with n = 4 in BSM and BLM as a function of the emittance normalized to the bucket area A (defined by Eq. (2.38)) compared to the single RF case. Right: the synchrotron frequency distribution in BLM (blue) and BSM (orange) are also compared in the case of a phase shift corresponding to 100 ps ($\Delta \phi_{800} = 28^\circ$). The voltage ratio $r = V_{800}/V_{200}$ is fixed to 0.1.



Figure 3.27 – Left: Synchrotron frequency spread at the SPS flat bottom energy (26 GeV/c) in a double RF system with a 200 MHz voltage of 4.5 MV for an emittance of 0.35 eVs as a function of the relative phase between the two RF systems ϕ_{800} . Right: Synchrotron frequency spread at the SPS flat top energy (450 GeV/c) in a double RF system with a 200 MHz voltage of 7.0 MV for an emittance of 0.5 eVs as a function of ϕ_{800} .

but as follows from Eq. (3.10), the synchrotron frequency in the bunch centre is not necessarily maximal [9]. By using a small perturbation $\Delta \phi = \phi_s - \phi_{s0} \ll 1$, the appropriate ϕ_{800} can be obtained using Eq. (2.50). The deviation from the synchronous phase in the single RF system during acceleration is [9]

$$\Delta \phi = \frac{r \tan \phi_{s0}}{n \cos \phi_{s0} - r},\tag{3.13}$$

and the relative phase in BSM is given by the expression

$$\phi_{800} = \pi - 4\phi_s - \arcsin\delta,\tag{3.14}$$

where $\delta = \frac{\sin \phi_{s0}}{-n \cos \phi_{s0} + r}$. For ϕ_{s0} close to π , the following approximation can be used

$$\arcsin \delta \approx \frac{\pi - \phi_{s0}}{4},$$
 (3.15)

as shown in Fig. 3.28 (left). Nevertheless, the difference between Eq. (3.12) and Eq. (3.14) is negligible in the SPS regarding the change in synchrotron frequency, as shown in Fig. 3.28 (right), since ϕ_s varies between π and 0.9π .

The reduction of the stability threshold due to the flat portion in the synchrotron frequency distribution in a double RF system will be examined in detail in Sections 4.2, 4.3, and 4.4.



Figure 3.28 – Left: Comparison of $\arcsin \delta$ (solid blue) in Eq. (3.14) and its approximation (dashed blue) in Eq. (3.15). Right: comparison of the approximated expression of ϕ_{800} (dashed blue) in Eq. (3.12) (blue) and the full expression (solid blue) in Eq. (3.14).

3.4.3 Controlled Longitudinal Emittance Blow-up

The coupled-bunch instability threshold is proportional to $\epsilon^{3/2}$, see Eq. (2.112). In the past, a controlled longitudinal emittance blow-up was used during the acceleration ramp in the Q26 optics, besides the 800 MHz RF system, to stabilize the LHC proton beam. However, the possible blow-up is limited by the maximum bunch length ($\tau = 1.9$ ns) accepted for injection into the 400 MHz RF system of the LHC without losses and satellites.

The controlled emittance blow-up is achieved using band-limited noise applied via the phase loop of the 200 MHz RF system [73]. It targets the desired bunch length by generating a noise in the band of the synchrotron frequency corresponding to the appropriate bunch emittance. The frequencies have to be corrected for the synchrotron frequency shift given by the machine reactive impedance (potential well distortion, see Section 2.5). This technique is difficult to apply in a single RF system due to the small synchrotron frequency spread. In a double RF system, it can also lead to non-uniform emittances along the batch in the presence of strong beam loading [50]. In recent operation, the controlled emittance blow-up was not necessary to ensure beam stability but it will be used for HL-LHC intensities.

The next section presents the last mentioned intensity limitation: the particle losses at injection and during the flat bottom.

3.5 Particle Losses

Losses of the LHC proton beam at the SPS injection and along the flat bottom have been observed and studied since the earliest days of the LHC operation [74]. They are increasing with intensity and can be a major bottleneck for the production of the HL-LHC beam. A degradation of the transmission by 20% has been observed at high intensity ($\sim 2.0 \times 10^{11}$ ppb) and the SPS RF upgrade alone would not be able to remove the losses [75]. These losses also

increase the intensity required from the SPS injector to an intensity that the PS would have problems delivering. A typical intensity measurement by BCT demonstrating the losses along the flat bottom and at the start of the acceleration is shown in Fig. 3.29 for a bunch intensity above nominal for the BCMS (Batch Compression, Merging, Splitting) [76] beam (48 bunches, operation).

The bunches in the PS are produced in a 40 MHz RF system. In addition, a 80 MHz RF system is also used. The RF voltage is not sufficient to reduce their length adiabatically and make them fit in the 200 MHz RF system of the SPS. A non-adiabatic procedure is applied to reduce sufficiently the length of the bunches. The 40 MHz and the 80 MHz RF voltages are raised sharply at the end of the cycle. The bunches rotate in the longitudinal phase space and can be extracted after a quarter of synchrotron period, when the bunch length is the smallest. This procedure is called bunch rotation. However, as it appears in Eq. (2.42), due to non-linearities of the RF voltage for long bunches, the outer part of the particle distribution in the longitudinal phase space lags and the bunch shape is distorted. To minimize this effect an 80 MHz RF system can be used beside the main 40 MHz RF system of the PS to perform the bunch rotation [29], but the non-linearities cannot be fully compensated. As shown in Fig. 3.30, the bunch distribution injected in the SPS has a peculiar shape called 'S-shape'. Bunch tails do not fully fit inside the SPS RF bucket, and they create uncaptured beam. Many particles are also injected close to the RF separatrix. After filamentation, they fill entirely the



Figure 3.29 – Measurement of intensity with a BCT from injection to the beginning of the ramp for a BCMS beam (blue) for 48 bunches ($N_b \sim 1.4 \times 10^{11}$ ppb). The momentum cycle (black) and the losses before (green arrow) and after the start of the acceleration (red arrow) are indicated. The 200 MHz voltage program is the LHC program (Fig. 3.4), the 800 MHz RF system was deactivated and the Q20 optics was used. The OTFB and the FF were activated.



Figure 3.30 – Phase space of the bunch in 200 MHz RF bucket at injection (26 GeV/c, Q20) (a) and after filamentation (b). A single RF system was used with $V_{200} = 4.5$ MV. The red line represents the RF separatrix.

SPS RF bucket, independent of the capture voltage [18].

The bunch population at large synchrotron amplitudes is too high to avoid losses due to any perturbations after injection and at the start of acceleration [6]. During the flat bottom, the intensity is decreasing continuously, see Fig. 3.29. Lost particles hit the momentum aperture, which is limited, especially for Q20 optics [49]. The remaining uncaptured beam is lost when the beam is accelerated, and since the RF bucket of the SPS is full after filamentation the ramp can never be completely adiabatic for all particles. At the start of acceleration, a sharp decrease in intensity is observed. The adiabaticity condition is defined by Eq. (2.69) but it cannot be fulfilled for $\omega_s \sim 0$. Due to beam loading and other perturbations (intensity effect, RF noise), particles close to the RF separatrix will be lost from the bucket. Projections for HL-LHC intensity suggest that the losses could exceed the LIU-SPS 10% loss budget for nominal longitudinal emittance [3]. From operational experience, the nominal LHC beam is currently at the limit of stability and a bunch emittance larger than the value of 0.4 eVs cannot be accelerated without an important part of the beam lost at the start of acceleration, see also Fig. 3.6 [7].

Beam measurements have been carried out on a 20 s long flat bottom (26 GeV/c) for a high intensity beam with 48 bunches. The standard 25 ns beam was compared with another type of beam called BCMS. Like for the standard 25 ns beam, the nominal bunch intensity of the BCMS beam is 1.15×10^{11} ppb, bunches are spaced by 25 ns for a longitudinal emittance of 0.35 eVs, but the total number of bunches in a SPS batch is 48. The BCMS beam allows having a smaller transverse emittance [49]. The losses are measured comparing the BCT at 200 ms after injection (after filamentation) and at the end of the flat bottom. The change in relative losses along the flat bottom cycle observed at the transition from the BCMS beam to the standard 25 ns beam are shown in Fig. 3.31. The first part of the measurements was



Figure 3.31 – Relative losses measured with a BCT for 48 BCMS bunches (first part of the measurements) and 48 standard 25 ns bunches (last part of the measurements) in a single RF system with a 200 MHz voltage of 4.5 MV at the flat bottom energy in the Q20 optic. Losses are obtained by comparing the BCT signals at $T_1 = 0.2$ s and $T_2 = 19.085$ s.

done with the BCMS beam with average losses of 5%. When the standard 25 ns beam was injected, the losses doubled. This strongly indicates that the transverse aperture limitation has a significant impact on the total losses.

Losses were also studied as a function of the 200 MHz voltage. A tune kick was applied in the beam gap after 2 s to remove the uncaptured particles from the ring. By comparing the BCT signal at 1.8 s and 2.1 s, this procedure allows measuring the amount of uncaptured beam. The total and continuous losses along the flat bottom are obtained by comparing the BCT signal at 50 ms and at 2.1 s respectively with the BCT signal at the end of the flat bottom. Results, as a function of the 200 MHz voltage, (constant during flat bottom) are shown for the BCMS beam in single RF system in Fig. 3.32. It can be observed that the amount of uncaptured beam is first decreasing when the voltage at 200 MHz is increasing. The acceptance is increased and more particles injected close to the RF separatrix are captured. However, the continuous loss along the flat bottom increases with the voltage. Particles are lost on the aperture due to the increase of momentum spread with larger 200 MHz voltage. The total transmission is affected by both contributions. It has a maximum around $V_{200} = 4.5$ MV. This corresponds to the capture and flat bottom voltage at 200 MHz used in the nominal acceleration cycle.

The beam loading at injection also has a significant effect on the population of the uncaptured beam. Simulations with particle distribution in the longitudinal phase space from Fig. 3.30 and OTFB and FF systems deactivated, show average losses in a batch of 72 bunches higher than 50%. Figure 3.33 shows particle losses along the batch at different moments after injection and the average value after the filamentation process. A bunch intensity of 1.4×10^{11} ppb was





Figure 3.32 – Relative losses measured by comparing the BCT signal at a different time for a high intensity ($N_b = 2.0 \times 10^{11}$ ppb) BCMS beam of 48 bunches. The OTFB system was activated. The voltage at 200 MHz was constant along the flat bottom and it is specified on each figure. The uncaptured beam is shown (a) together with the losses during the flat bottom (b). The total losses at the end of the flat bottom (26 GeV/c) are also presented (c).

used and the beam was injected in a single RF system with $V_{200} = 4.5$ MV, constant along flat bottom. The losses are growing along the batch. This indicates that the beam loading plays a significant role in the loss mechanism. The e-cloud effect could also be involved but this hypothesis has been excluded [49]. A beam pattern called 8 bunches 4 empty (8b4e), which practically eliminates the e-cloud effect, has been compared to the 25 ns nominal beam. The losses were comparable, with the same loss pattern along the batch in both cases [77].

When the OTFB and FF systems are activated, the transient beam loading is reduced as well as the uncaptured beam. It has been determined in simulations that a significant part of the losses observed is created during the bunch-to-bucket transfer and that the amount of uncaptured beam from injection is fixed after half a synchrotron period ($\sim 0.7 \text{ ms in Q20}$). The simulations were done for 72 bunches in a double RF system at flat bottom with a 200 MHz voltage of 4.5 MV and a voltage ratio between the two RF systems of 0.1. The bunch distribution in the SPS after bunch rotation in the longitudinal phase space was used and the bunches were centred in the RF bucket since the phase loop, which would centre the RF wave on the



Figure 3.33 – Particle losses in simulations of 72 bunches at the flat bottom energy in a double RF system with a 200 MHz voltage of 4.5 MV and a voltage ratio of 0.1. The particle distribution in phase space at injection is the one after bunch rotation in the PS (see Fig. 3.30). The bunch intensity is $N_b = 1.4 \times 10^{11}$ ppb. The OTFB and FF systems are deactivated. Losses were measured by comparing the number of particles outside the RF bucket (with intensity effects) and the total number of particles in the simulation. The blue curve corresponds to the losses at injection. The green and red curves correspond to the losses at a cycle time t = 96 ms and t = 231 ms respectively. The dashed (red) line corresponds to the average loss at a cycle time t = 231 ms.

bunches in operation, is not included in simulations. A lower impedance reduction by the OTFB and the FF systems (<20 dB) during the transient time increases the number of lost particles.

The two efficient mitigation measures for the particle losses would be either to inject smaller bunches from the PS if beam stability allows it, or to use in the SPS an additional capture RF system with a lower harmonic number to increase the size of the RF bucket. The latter option is treated in the last chapter of this thesis.

3.6 Conclusion

In this Chapter, the three main intensity limitations in the longitudinal plane, which pose serious challenges for the intensity increase in the SPS, have been analysed together with the existing mitigation measures.

The first one, exposed in Section 3.1, is the beam loading in the 200 MHz RF system, which is the result of the large impedance at the fundamental frequency of the cavities. The corre-

sponding induced voltage can reduce significantly the voltage available for the beam which may lead to lack of voltage at flat top required to ensure the beam stability and to particle losses during acceleration. The beam loading can also create RF amplitude and phase errors upon beam injection, which cause uncontrolled emittance blow-up. It also induces significant power loss, which shifts the synchronous phase along the batch and makes the phase control of the double RF operation more complicated.

The second main intensity limitation is due to longitudinal beam instabilities. The complete longitudinal impedance model of the SPS has been described in Section 3.2. The existence of this model was a key component in the understanding of the beam instabilities in the SPS. In Section 3.3, we have verified that the existing SPS impedance model allows the reproduction in simulations of the single-bunch instabilities. This also justifies the study of the multi-bunch instabilities with this model. We have determined through dedicated beam measurements that the instability observed at flat top is a superposition of dipole and quadrupole modes. Using the growth rate of instability explained in Section 2.8, we have found that the instability observed is most likely due to the 630 MHz HOM of the 200 MHz RF TW structures. The stability thresholds of 72 bunches at the flat top energy have been simulated and the results confirm the present measured threshold. In a double RF system, our simulations show that the stability threshold is not only affected by this HOM but also by the impedance of the QF vacuum flanges.

For the LHC beam, the average bunch length along the batch at flat top is limited to 1.65 ns for injection into the 400 MHz RF bucket of the LHC. The stability threshold in a single 200 MHz RF system is three times below the nominal bunch intensity and mitigation measures are necessary; they have been examined in Section 3.4.1. To reduce the effect of the beam loading, the 200 MHz RF system is equipped with an OTFB and a FF system, which reduces the beam-coupling impedance at 200 MHz by -20 dB. However, due to the limited RF power, the RF voltage available for the beam at flat top strongly depends on the bunch intensity in present operation but also after the RF upgrade. This limitation restricts the bunch emittances that can be accelerated in the SPS to the flat top energy as shown in Fig. 3.6. We have verified that an increase of the transition energy could also reduce the power demand during acceleration since the voltage needed for a given emittance becomes smaller. However, at flat top, because of the beam loading and the restriction of the bunch length, the stability thresholds are similar for the three different optics, which was validated in simulations.

We also studied, in Section 3.4.2, the operation of the double SPS RF system, which improves the beam stability. This system increases the synchrotron frequency spread within bunches which enhances the Landau damping of bunch instabilities. We have confirmed that the bunch-shortening mode is preferable in the SPS in comparison to the bunch-lengthening mode because the flat portion of the synchrotron frequency distribution is further away from the bunch centre. The bunch-shortening mode is also easier than the bunch-lengthening mode to use in operation since the synchrotron frequency spread depends less strongly on the relative phase between the two RF systems (affected by the beam loading).

The third major intensity limitation, analysed in Section 3.5, is the particle losses which are increasing with intensity. They are observed at the flat bottom energy and at the start of the acceleration, mainly due to the particle distribution at injection into the SPS. The bunch rotation in the PS creates a S-shape in the particle distribution in the longitudinal phase space and the SPS 200 MHz RF bucket is full after the filamentation, independent of the RF voltage at injection. However, we have confirmed, by dedicated beam measurements, that the optimal value of this voltage to minimize particle losses is 4.5 MV in the Q20 optics. Beam measurements have also indicated that without mitigation measures the particle losses will exceed the LIU loss budget of 10%. To reduce the losses, bunches with a smaller emittance could be injected into the SPS if the beam stability allows it in the PS, or an additional RF system for the bunch capture could be installed in the SPS, which is the subject of Chapter 6.

4 Longitudinal Multi-Bunch Instabilities in the Present SPS

The multi-bunch instabilities developing in the SPS during the acceleration cycle are a severe intensity limitation. It is necessary to know the causes of these instabilities to find mitigation measures and implement them to reach higher bunch intensity. Particle tracking simulations can be used to identify the effect on beam stability of different contributions to the longitudinal impedance model of the SPS. Simulations also allow studying the beam stability in conditions after LIU upgrades, not reachable yet in beam measurements. However, to validate the predictions of simulations, beam measurements are necessary to benchmark the existing longitudinal impedance model. Beam measurements and particle simulations are also used to assess the effect on beam stability of the 200 MHz LLRF system and the double RF operation.

In this chapter the results of studies of longitudinal multi-bunch instability by means of beam measurements are analyzed and compared to analytical estimations and simulations. A single batch of 12 bunches spaced by 25 ns was chosen to study the instabilities during the acceleration cycle, since in this case the beam loading is reduced as compared to the nominal case with 72 bunches, the RF system is not power limited in the bunch intensity range of interest and a batch (sometimes unstable) with a bunch intensity up to $N_b = 1.5 \times 10^{11}$ ppb can be accelerated to flat top (450 GeV/c) in a single 200 MHz RF system without the OTBF (oneturn-delay feedback) and FF (feedforward) systems. A bunch intensity of 2.5×10^{11} ppb was also reached with the OTFB and FF systems activated. This beam allowed studying the beam stability as a function of the bunch intensity in controlled conditions (no power limitations). In a single RF system with the beam considered, unknowns difficult to monitor in real machine conditions or difficult to reproduce in simulations, e.g. the relative phase between the two RF systems or the controlled emittance blow-up, are removed. In all simulations, the phase loop, which centres the bunches in the RF bucket and is always activated in beam measurements, is not included and the position of the bunches is initially matched to the RF bucket including intensity effects to reproduce its effect.

As a second step, this beam was used to study the beam stability in the double RF operation. For beam measurements in a double RF system, the relative phase between the two RF systems was calibrated at the beginning of the run to follow Eq. (3.12) during cycle. The calibration



Figure 4.1 – Bunch profile (blue) and potential well (black) in a double RF system for n = 4 in bunch-shortening mode, i.e. $\phi_{800} = \pi$ (a), and in the case with an offset in the phase with respect to the BSM (b). The vertical dashed lines indicate the bucket center in the 200 MHz RF system. The voltage ratio between the two RF system is r = 0.1.

of the relative phase between the two RF systems, ϕ_{800} is made by measuring the tilt on the profile of a single bunch. As illustrated in Figure 4.1, when the phase ϕ_{800} deviates from its value in bunch-shortening mode the bunch profile is asymmetric and by varying ϕ_{800} , it is possible to find the correct value of ϕ_{800} corresponding to the bunch-shortening mode. The calibration was made at flat top and flat bottom and ϕ_s is defining ϕ_{800} during the ramp according to Eq. (3.12). The two systems are, therefore, assumed to be in the bunch-shortening mode.

In the first part of this chapter, the beam stability through the ramp in a single RF system (200 MHz) is discussed. The effects on the stability thresholds of the OTFB and the FF systems are also analysed. The longitudinal impedance model of the SPS is benchmarked by comparing measured stability thresholds with those simulated through the ramp or only at flat top. Results of measurements of stability thresholds with smaller longitudinal emittance are also shown. The effect of the 200 MHz fundamental impedance in the longitudinal impedance model of the SPS is also investigated in simulations. The analytical estimation of the coupled bunch instability threshold, $N_{\rm th}^{\rm CBI}$ defined by Eq. (2.112), is compared to measurements with and without OTFB system. The stability threshold during the acceleration cycle in a double RF system is analysed. The effect of the voltage ratio on the stability threshold is studied and an optimized voltage ratio program enhancing beam stability through cycle is obtained.

4.1 Stability Threshold Through Ramp in a Single RF System

In beam measurements, the nominal LHC cycle with the momentum program shown in Fig. 1.2 was used with only the first injection at flat bottom. The beam stability was studied in the Q20

optics with the nominal RF voltage program (Fig. 3.4). The stability threshold was obtained like in simulations, using the value of the maximum amplitude of the bunch length oscillations during cycle $\Delta \tau$, defined by Eq. (3.6), to separate stable beam from unstable. When $\Delta \tau$ exceeds 0.07 at a cycle time t_0 , the beam was considered as unstable and $E_s(t_0)$ is the energy at which it became unstable.

The largest effect on the stability threshold during acceleration was observed to be from the action of the OTFB and the FF systems. The stability thresholds as a function of the synchronous beam energy E_s are shown in Fig. 4.2 for the OTFB system on and off. The bunch intensity is the average bunch intensity at flat bottom after filamentation. Thresholds measured when the OTFB was deactivated (blue) are distinguished from those where it was active (red). In the case without OTFB, the FF was also deactivated. Measurements with or without the FF, when the OTFB was activated, did not exhibit any significant difference in the stability threshold and are not shown. The effect of the longitudinal damper will be discussed in Section 4.1.1, but it was not included in simulations, since it did not contribute to the stability threshold for 12 bunches. The effect of the OTFB is addressed first in the next section.

4.1.1 Stability Threshold Without One-Turn-Delay Feedback

For the LHC beam in the SPS, the stability threshold of the coupled-bunch instability $N_{\text{th}}^{\text{CBI}}$ defined by Eq. (2.112) decreases during acceleration as it depends on the inverse of the syn-



Figure 4.2 – Measured instability thresholds during the ramp as a function of the beam energy E_s , in the cases without OTFB (blue) and with OTFB (red), in a single RF with the nominal voltage program of the LHC cycle (Fig. 1.2). The longitudinal bunch emittance at injection was the nominal 0.35 eVs.

chronous energy E_s . The results of measurements presented in Section 3.3 suggest that the 630 MHz HOM impedance of the 200 MHz TW cavities is one of the main causes of the instability observed at flat top. Figure 4.3 shows the analytic estimation of the stability threshold from Eq. (2.112), compared to the threshold measured when the OTFB was deactivated.

The analytic threshold is computed for a resonant frequency $f_r = 630$ MHz and impedance $R_{\rm sh} = 570$ k Ω corresponding to the HOM, assuming a ring filled with equidistant bunches at 25 ns. The bunch parameters (energy spread, synchrotron frequency spread, bunch length) are computed at every revolution, for a particle distribution in phase space matched to the RF bucket without intensity effects. This estimation agrees with measurements from the middle of the acceleration cycle to flat top. However, this agreement has to be taken with caution. The analytical estimation assumes a full ring with bunches spaced by 25 ns (924 bunches), which is therefore the lowest limit, but the effect of the different particle distribution can easily shift up the stability threshold by a factor two [48]. Moreover, during acceleration, the injected bunch emittance (0.35 eVs) can only increase and the matched bunch emittance at flat top (7 MV) for a 4- σ bunch length of 1.65 ns is 0.47 eVs (computed numerically using Eq. (2.70)) in the Q20 optics which would give a higher threshold than the one computed with 0.35 eVs during the cycle.



Figure 4.3 – Measured instability thresholds during ramp as a function of the beam energy E_s , in the case without OTFB and FF systems, in a single RF system with nominal voltage program. The longitudinal emittance at injection was the nominal 0.35 eVs. The analytic estimation of the threshold using Eq. (2.112) (orange), computed for an emittance of 0.35 eVs, with bunch parameters matched to the RF bucket at every moment for the 630 MHz HOM with $f_r = 630$ MHz and $R_{\rm sh} = 570$ k Ω . A single RF system was used (200 MHz) with the nominal voltage program shown in Fig. 3.4.

Nevertheless, it is interesting that the estimation of the instability threshold suggests two different instability regimes in the beam measurements. In the first part of the ramp, the analytical estimation of $N_{\rm th}^{\rm CBI}$ for the 630 MHz HOM is far above the measured intensity limit. It means that most likely another instability is dominating at flat bottom and during the first part of acceleration, which will be treated in Section 4.1.2.

To verify this hypothesis, particle simulations were carried out for the full SPS impedance model using matched particle distributions at flat bottom with different emittances and results are shown in Fig. 4.4. The bunch length at flat bottom, after filamentation (~100 ms), in simulations is $\tau_{\rm fil}$, to be compared with those measured at the same time. The bunches were matched to the RF bucket with intensity effects. A reasonable agreement is obtained between measurements and simulations for bunches which had the same bunch length at flat bottom, from the middle of the acceleration cycle until flat top. In the first part of the ramp, however, the beam is more stable in simulations than in measurements. The difference can be due to the particle distributions in phase space after the bunch rotation in the PS, which plays a significant role in the formation of instabilities at flat bottom. This effect is confirmed in Section 4.3 for a double RF system, where instabilities at low energy (26 GeV) have been



Figure 4.4 – Measured and simulated instability thresholds during the ramp as a function of the beam energy E_s , in the case without the OTFB system, in a single RF system with the nominal voltage program. The colours represent the bunch length at flat bottom after filamentation in the 200 MHz voltage of 4.5 MV. Simulations are for bunch lengths at flat bottom of $\tau_{fil} = 2.60$ ns (0.27 eVs) (green), $\tau_{fil} = 2.75$ ns (0.3 eVs) (blue), and $\tau_{fil} = 3.05$ ns (0.35 eVs) (red). Results of simulations are linearly interpolated between the points. In simulations, the particle distribution in phase space was matched at flat bottom to the RF bucket without intensity effects. The longitudinal impedance model of the SPS was used.

studied in simulations including bunch rotation in the PS.

The instability threshold through ramp was also measured with a longitudinal bunch emittance smaller than nominal. Figure 4.5 shows the instability threshold for $\epsilon = 0.25$ eVs compared to the analytic estimation of the threshold of coupled bunch instability caused by the 630 MHz HOM. A reasonable agreement between the beam measurements and the analytic estimation can be obtained only after the start of the ramp which is another indication of the predominance of another impedance or the bunch distribution in the longitudinal phase space (or both), in the onset of instability during the earlier part of the cycle. One can see that the threshold is lower than for 0.35 eVs.

The effect of the longitudinal damper (LD) [1] on the stability threshold was also investigated in beam measurements. The LD is a part of the LLRF system that damps the dipole oscillations of the batch as a whole. For corrections, it uses the 200 MHz RF cavities and therefore is limited by its bandwidth. The OTFB was deactivated for this study. Stability thresholds when the LD was activated (green) and deactivated (purple) shown in Fig. 4.6 are very similar. This is why its effect can be neglected in simulations of 12 bunches; beam measurements with and without LD were also not distinguished previously.

To conclude this section, simulations, measurements, and analytic estimation agree well



Figure 4.5 – Measured and simulated instability thresholds during the ramp as a function of the beam energy E_s , in the case without the OTFB system, in a single RF system with the nominal voltage program, for a longitudinal bunch emittance $\epsilon = 0.25$ eVs, smaller than nominal. The analytical estimation of the threshold from Eq. (2.112) (orange curve) is computed for the 630 MHz HOM and an emittance of 0.25 eVs with bunch parameters matched to the RF bucket at every moment.



Figure 4.6 – Instability thresholds during ramp, as a function of the beam energy E_s , in the cases with and without the longitudinal damper and the OTFB off. The longitudinal bunch emittance at injection was nominal (0.35 eVs). A single 200 MHz RF system was used with the nominal voltage program (Fig. 3.4).



Figure 4.7 – Stability thresholds simulated at flat top only (blue) are compared with results of simulations through the ramp (orange). The measurements for bunches which were stable at a flat top are also indicated (green dots). The OTBF and the FF systems were off.

during the second part of the acceleration cycle. This is a strong indication that the 630 MHz HOM is predominant in the development of the multi-bunch instability, not only at flat top, but also during acceleration. Moreover, this is also a good benchmark of the longitudinal impedance model of the SPS. In Fig. 4.7, the stability thresholds simulated only at flat top are compared to the stability thresholds simulated through the acceleration ramp with bunches matched to the RF bucket at flat bottom and the measured instability thresholds. Simulations and beam measurements agree, regarding the stability threshold at flat top. This supports the validity of simulations at flat top for the situation after LIU upgrades which are presented in Chapter 5. However, particle simulations at flat bottom have difficulties to reproduce the measured instability threshold. This fact is due to the bunch distribution at injection which is not matched to the RF bucket as it will be explained in Section 4.3. The stability threshold during acceleration with the OTFB and the FF systems on are discussed in the next section.

4.1.2 Stability Threshold With One-Turn-Delay Feedback

When the OTFB and the FF systems are activated the stability threshold during the acceleration cycle changes significantly. Figure 4.8 shows the instability thresholds measured with the OTFB and the FF systems on, compared to the analytic estimation of the threshold for the



Figure 4.8 – Measured instability thresholds during the SPS ramp, as a function of the synchronous energy E_s , in the case with the OTFB and the FF systems on. The analytical estimation of the threshold from Eq. (2.112) is computed for the nominal emittance of 0.35 eVs, with bunch parameters matched to the RF bucket at every moment, assuming a 630 MHz resonator impedance with $R_{\rm sh} = 570 \text{ k}\Omega$. A single 200 MHz RF system is used with the nominal voltage program (Fig. 3.4). The stability thresholds measured with the OTFB deactivated are also indicated (light blue).

630 MHz HOM. The measurements without OTFB are also indicated on the plot. The instability threshold follows the same trend at the end of the acceleration compared to the case without the OTFB and the FF systems. The OTFB cannot mitigate the instability if it is due to the 630 MHz HOM, but it does reduce the 200 MHz impedance and the beam loading. At flat bottom, for this set of measurements with 12 bunches in a single RF system, only stable beams were observed, they become unstable later in the cycle. This fact suggests that the instability observed at lower energy is caused by the fundamental impedance of the 200 MHz RF system.

At intensities higher than 1.4×10^{11} ppb, the beam becomes unstable at energies between 100 and 300 GeV with no visible energy dependence of the threshold, even though it would be expected from Eq. (2.112). This absence of energy dependence can be explained by an increase in the bunch emittance with the intensity. Figure 4.9 shows the beam measurements, when the OTFB was on, with colours indicating the bunch length at flat bottom, after filamentation. The 200 MHz voltage at capture is the same for all the measurements (4.5 MV). However, for bunch intensity above 1.5×10^{11} ppb, the bunch length at given energy E_s is gradually increasing with intensity. This could be due to the fact that injected bunches have a larger longitudinal emittance for higher intensity. This effect can be explained by the uncontrolled emittance



Figure 4.9 – Measured and simulated instability thresholds during ramp, as a function of the beam energy E_s , in the case with the OTFB and the FF systems on, in a single RF system with the nominal voltage program. The colours represent the bunch length at flat bottom after filamentation with a 200 MHz voltage of 4.5 MV. Simulations are for bunch lengths at flat bottom of $\tau_{\rm fil} = 2.60$ ns (0.27 eVs) (green), $\tau_{\rm fil} = 2.75$ ns (0.3 eVs) (blue), and $\tau_{\rm fil} = 3.05$ ns (0.35 eVs) (red). Effects of the OTFB and the FF systems are included in simulations using Eq. (3.7). Results of simulations are linearly interpolated between the points. In simulations, the particle distribution in phase space was matched to the RF bucket, at flat bottom, including intensity effects. The full longitudinal impedance model of the SPS was used.

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blow-up in the PS. The stability threshold is higher for larger bunch emittance, as it appears in the analytical estimation in Eq. (2.112), and this increase compensates the decrease of the stability threshold with the beam energy. As a result, the stability threshold does not exhibit clear energy dependence. Simulations were carried out to confirm this hypothesis. Batches of 12 bunches with a binomial distribution defined by Eq. (2.108), with $\mu = 1.5$, were matched at flat bottom. The nominal voltage program was used for the 200 MHz RF system. The effect of the OTFB and the FF systems is modelled in simulations using Eq. (3.7). Results are included in Fig. 4.9. The simulated threshold is increasing for larger bunch length after filamentation, τ_{fil} , following the measurements. However, as shown later in Section 4.3, to obtain better agreement for the first part of the ramp, realistic particle distribution in the longitudinal phase space, after bunch rotation in the PS, should be used in simulations.

To summarize, the beam measurements with the OTFB activated confirm the hypothesis of the two different sources of instability during SPS ramp. The coupled-bunch instability observed at the end of the acceleration ramp and at flat top is most probably caused by the 630 MHz HOM of the 200 MHz RF system. At flat bottom and in the first part of the acceleration, another instability occurs, which is determined by the fundamental impedance of the 200 MHz cavities.



Figure 4.10 – Measured and simulated instability thresholds for 12 bunches during ramp, as a function of the beam energy E_s , with the OTFB and the FF systems on, in a single RF system with nominal voltage program. The colours represent the average bunch length measured at 11 s (end of flat bottom). Simulations are for a bunch length at flat bottom $\tau_{fil} = 2.75$ ns and the SPS impedance model was used with the fundamental impedance of the 200 MHz RF system removed from the model (solid blue) and the 200 MHz and the 630 MHz HOM removed (dashed blue). Results of simulations are linearly interpolated between the points. The particle distribution was matched to the RF bucket with intensity effects at flat bottom.

However, the effect of this impedance is not necessarily simple to model. Figure 4.10 shows the simulated instability threshold during acceleration of 12 bunches matched to the RF bucket (including intensity effects) at flat bottom. The longitudinal impedance model presented in Section 3.2 was used in simulations, where the fundamental impedance of the 200 MHz cavities was removed (solid blue). This case is also compared to the case when the fundamental impedance of the 200 MHz cavities and the HOM at 630 MHz were both removed. When the beam loading is absent (no 200 MHz impedance), the beam is more unstable in simulations. When also the impedance of the HOM at 630 MHz is removed from the longitudinal impedance model, the stability threshold increases. One explanation of this beneficial effect from the beam loading is that it introduces synchrotron frequency modulation from bunch to bunch which decouples them within the batch [51]. The presence of beam loading improves the beam stability for a batch of 12 bunches. This is why a good model of the OTFB and the FF systems is essential for accurate simulations.

To conclude this section, Fig. 4.11 (left) shows the average bunch length at the flat bottom energy and the flat top energy within 12 bunches as a function of the average injected bunch intensity, for the case with the OTFB and the FF systems on. The error bars represent the maximum amplitude of the bunch length oscillations within the beam at flat bottom and flat top. Large error bars indicate large bunch length oscillations within the beam. For a bunch intensity $N_b \ge 0.5 \times 10^{11}$ ppb the beam becomes unstable during the ramp and is stable at flat bottom. The relative losses from injection to flat top (normalized to the injected bunch intensity) are also shown in Fig. 4.11 (right). They increase with intensity and reach ~16% at $N_b = 2.4 \times 10^{11}$ ppb. In comparison, in a double RF system, treated in the next section, the beam stability will be significantly improved but the beam losses will be comparable.



Figure 4.11 – Left: average bunch length for 12 bunches at flat bottom measured after 100 ms and at flat top as a function of the average bunch intensity injected. The injected longitudinal bunch emittance was nominal (0.35 eVs). The 200 MHz voltage at flat bottom was 4.5 MV and 7.0 MV at flat top, and the OTFB and the FF systems were on. Right: particle losses measured from injection to flat top as a function of the bunch intensity injected N_b . The losses are normalized to the average bunch intensity at injection.

4.2 Stability Threshold Through Ramp in a Double RF System

The beam measurements in a single RF system allowed the effects of the different parts of the 200 MHz LLRF system on beam stability to be investigated. They also allowed the SPS longitudinal impedance model to be benchmarked and the validity of predictions made at flat top to be justified. The fourth harmonic RF system is the main mitigation measure against beam instabilities in the SPS. Beam measurements were carried out to find possible ways of improving beam stability. For measurements in the double RF system, all of the 200 MHz LLRF beam control were activated. Recently, after the upgrade of the 800 MHz RF system, it was also equipped with an OTFB system which reduces the beam loading in the 800 MHz TW cavities, helping the control of the relative phase between the two RF systems. We do not discuss its effect here but it will be shown in Chapter 5 that it can increase the stability threshold on the flat top. The controlled emittance blow-up was not used.

The second RF system in the SPS improves the Landau damping by increasing the synchrotron frequency spread, as suggested by the analytical estimation of the coupled-bunch instability threshold in Eq. (2.112). The increase of synchrotron frequency in the bunch centre is given by Eq. (2.47). However, as explained in Section 3.4.2, a drastic reduction of the stability threshold can be observed when the derivative with respect to the action (similarly, the emittance) of the synchrotron frequency distribution goes to zero within the bunch [35]. In the SPS, this situation differs between flat bottom and flat top.

To explore the beam stability in the double RF system and study possible ways of increasing further the synchrotron frequency spread, in beam measurements the voltage ratio between the two RF systems was varied for different injected bunch intensities from $N_b = 1.4 \times 10^{11}$ to $N_h = 2.45 \times 10^{11}$ ppb. The nominal LHC momentum cycle was used as well as the nominal 200 MHz RF voltage program and the voltage ratio during the acceleration cycle $V_{800}/V_{200} = r$ was varied in the range (0.05, 0.25). Figure 4.12 shows examples of the bunch length measured during the cycle in the double RF system with a voltage ratio between the two RF systems $V_{800}/V_{200} = 0.1$ (left) and $V_{800}/V_{200} = 0.25$ (right). The voltage ratio was constant during the cycle. The average bunch length of 12 bunches is presented with the maximum and minimum values in the batch. The longitudinal bunch emittance was nominal (0.35 eVs) and the bunch intensity at injection was $N_b = 2.45 \times 10^{11}$ ppb. In the case of the nominal (used in operation) voltage ratio $(V_{800}/V_{200} = 0.1)$, the maximum and minimum bunch length follows the average value during the acceleration cycle. However, at the end of the cycle, when the beam arrives at flat top, the average bunch length increases and the maximum bunch length along the batch deviates from the average. This fact indicates that the beam becomes unstable. When the voltage ratio is increased to $V_{800}/V_{200} = 0.25$, the beam is stable, and the bunch length does not oscillate at flat top. This effect has been observed on a large sample of measurements, also with smaller bunch intensities. It shows that a batch of 12 nominal LHC bunches can be stabilized at flat top by increasing the voltage ratio between the two RF systems. Nevertheless, with a voltage ratio of 0.25, constant during the cycle, at flat bottom the bunch length within the batch exhibits a large spread and the average bunch length is slightly increasing. This is also observed



Figure 4.12 – Average (blue) and maximum/minimum (red) bunch length measured during the acceleration cycle for a batch of 12 bunches. The minimum and maximum bunch length along the batch are also shown. The double RF system of the SPS was used with $V_{800}/V_{200} = 0.1$ (left) and $V_{800}/V_{200} = 0.25$ (right). The x-axis is labelled in terms of the timestamp of the acquisition. Measurements were acquired every 50 ms during the cycle, and the data is linearly interpolated between the points.

at different bunch intensities and for different voltage ratios between the two RF systems larger than nominal 0.1. The amplitude of the bunch length oscillations increases at flat bottom when the voltage ratio increases and the beam becomes more unstable. An example of the evolution of the bunch profile measured during the cycle, for $V_{800}/V_{200} = 0.25$, is shown in Fig. 4.13. The bunch length oscillations observed at the flat bottom energy are damped at the end of the acceleration but the unstable behaviour of the beam at flat bottom impacts the beam quality (larger tails) and losses are possibly increased at the start of acceleration. Use of a smaller voltage ratio (0.1) at flat bottom can cure these oscillations.

For a RF harmonic ratio n = 4, as in the SPS, for voltage ratios in BSM above some critical value, a plateau can appear in the synchrotron frequency distribution, where the derivative is close to zero inside the bunch, induced by the fourth harmonic or the intensity effects. Particles in this region may develop a large coherent response as shown in Section 2.7.

At flat bottom some particles within the nominal longitudinal emittance are contained in a region where the derivative of synchrotron frequency $\omega'_s(J)$ is close to zero and the stability threshold is reduced. When the synchrotron frequency derivative goes to zero, the Landau damping is lost and instabilities can be triggered by any perturbation. However, this mechanism depends on the number of particles stored in the region in the longitudinal phase space where the synchrotron frequency derivative goes to zero, ΔN_b . We define the parameter S as follows

$$S(J) = \frac{\Delta N(J)}{\omega'_{S}(J)}.$$
(4.1)

When $\omega'_{s}(J)$ goes to zero (or becomes very small) but *S* does not increase, meaning that the number of particles within this region is very limited, an instability will not appear. On the



Figure 4.13 – Evolution of the measured profile of the bunch 7 within a batch of 12 bunches in the double RF system with voltage ratio $V_{800}/V_{200} = 0.25$. The injected bunch intensity was 2.45×10^{11} ppb. The profiles are shown at different cycle times *t* during the nominal acceleration cycle (Fig. 3.4). The time *t* = 10.5 s (blue) corresponds to flat bottom and *t* = 19.9 s (purple) to flat top.

contrary, if $\omega'_s(J) \ll 1$ and S(J) increases when approaching this region, it is likely that an instability will be triggered. The value of the parameter *S* and its relation with the instability was not studied, here we focus on the intensity limit as a function of the bunch length.

At flat top, the derivative of the synchrotron frequency does not go to zero within the bunches with the nominal emittance at injection of 0.35 eVs and significant improvements of beam stability can be obtained with a larger voltage ratio. This also explains why BLM cannot be used for beam stabilization in the SPS. Indeed, there the region with $\omega'_{s} = 0$ exists, independent of *r*. The normalized synchrotron frequency distribution as a function of the longitudinal emittance (normalized to the bucket area A) is shown in Fig. 4.14 for different voltage ratios. Increasing r increases the synchrotron frequency spread within the bunch outside the plateau, but only for short bunches. Indeed, at flat top, bunches with nominal longitudinal emittance are far from the plateau of the synchrotron frequency distribution. At flat bottom, however, the bunch contains a region where the synchrotron frequency distribution becomes flatter with increasing voltage ratios. In the case of a derivative with small non-zero values, the stability threshold is reduced, like in the case where the derivative goes to zero. The minimum of the synchrotron frequency derivative for different voltage ratios is shown in Fig. 4.15 for longitudinal emittances $\epsilon \le 0.35$ eVs (left) and $\epsilon \le 0.6$ eVs (right). The case with a longitudinal emittance of 0.6 eVs corresponds to the average matched emittance, at flat top, after LIU upgrades (0.57 eVs). At flat top, the voltage ratio can be safely increased, but at flat bottom



Figure 4.14 – Normalized synchrotron frequency in BSM as a function of the relative longitudinal emittance normalized to the bucket area *A*. The single and the double RF system cases with different voltage ratios are shown. The dashed vertical lines correspond to the longitudinal emittances at flat top (left line) and flat bottom (right line). The 200 MHz voltage at flat bottom is $V_{200} = 4.5$ MV and at flat top $V_{200} = 7.0$ MV.



Figure 4.15 – Minimum of the normalized synchrotron frequency derivative within bunches with nominal longitudinal emittance 0.35 eVs (left) and $\epsilon = 0.6$ eVs (right) as a function of the voltage ratio in a double RF system in BSM at the flat top and the flat bottom energy.

the derivative inside the bunch is decreasing with V_{800}/V_{200} and approaches zero for ratios $V_{800}/V_{200} > 0.20$.

The effect of the voltage ratio on beam stability is studied in Section 4.3 at flat bottom and in Section 4.4 at flat top, using beam measurements and simulations. An optimized voltage ratio program, during full acceleration cycle is also obtained. The latter has been successfully

tested in SPS operation, and the beam stability was improved.

4.3 Effect of 800 MHz RF System on Beam Stability at the SPS Flat Bottom Energy

The beam stability in a double RF system was studied on flat bottom using the LHC cycle voltage program. Figure 4.16 (left) shows the maximum amplitude of the bunch length oscillations of 12 bunches at flat bottom normalised by the average bunch length measured at the end of flat bottom (11 s) as a function of the average injected bunch intensity. The cases with $V_{800}/V_{200} = 0.10, 0.15, 0.20, \text{ and } 0.25$ are presented. When the voltage ratio increases from the nominal 0.1, the bunches become more unstable in the intensity range of the measurements. However, particle losses at the start of the acceleration are not particularly increasing with the voltage ratio. Figure 4.16 (right) shows the relative losses between injection and flat top as a function of the average injected bunch intensity. Losses are increasing with the bunch intensity but not with the voltage ratio. They are also comparable to the single RF case (see Fig. 4.11 (right)).

When the OTFB and the FF systems were deactivated, a beam instability has been observed at flat bottom for intensities above nominal. This instability is likely caused by the fundamental impedance of the 200 MHz RF system which will be further reduced after the planned LLRF upgrade. Nevertheless, if another source of impedance contributes to this instability, the 800 MHz RF system will lack efficiency to mitigate it, since after the capture of rotated bunches in the PS, the SPS RF bucket is full [18]. With the OTFB and the FF systems activated at flat



Figure 4.16 – Left: Measured maximum relative amplitude of the bunch length oscillations of 12 bunches during flat bottom as a function of the average injected bunch intensity N_b . Right: Corresponding particle losses between injection and flat top as a function of the average injected bunch intensity N_b . The losses are normalized to the average bunch intensity at flat bottom. The injected bunch emittance was nominal (0.35 eVs). The cases $V_{800}/V_{200} = 0.10$ (blue), 0.15 (green), 0.20 (grey) and 0.25 (red) are presented. The 200 MHz voltage program is the LHC cycle from Fig. 3.4, and the OTFB and the FF systems were on.
bottom, it was also observed that a voltage ratio of 0.1 provides better stability than a larger value of the ratio for batches of 48 bunches with intensities above nominal.

To remove the plateau in the synchrotron frequency distribution it is also possible to shift the relative phase ϕ_{800} slightly away from the bunch shortening mode. Improvements of the stability with a phase shift have been demonstrated in simulations in the past [78]. However, the longitudinal acceptance is also reduced in this case, which may lead to additional particle losses.

The stability thresholds measured for batches of 12 bunches without the OTFB nor the FF systems are compared in Fig. 4.17 with simulations. First simulations have been carried out with bunches matched to the RF bucket (with intensity effects) in a single RF system. The maximum amplitude of the bunch length oscillations during cycle (normalized by the average) was used as a criterion to separate stable from unstable beams, similarly to measurements. The stability limit was, however, far above the measured one. Much better agreement is obtained when the bunch rotation in PS is included in simulations. This indicates that the realistic particle distribution, defined by the PS, has an important effect on the instability occurring during the SPS flat bottom. Particles fill the RF bucket after filamentation and resulting bunch profiles interact more with the high frequency part of the longitudinal impedance



Figure 4.17 – Stability threshold at flat bottom as a function of the bunch length after filamentation for a batch of 12 bunches matched to the RF bucket with intensity effects. The 200 MHz voltage is 4.5 MV, r = 0 (single RF), and the OTFB and the FF are deactivated. The full SPS longitudinal impedance model is used. Corresponding beam measurements are included for comparison. For simulations, colours correspond to the maximum amplitude of the bunch length oscillations during the cycle, normalized by the average.

Chapter 4. Longitudinal Multi-Bunch Instabilities in the Present SPS

of the machine. The particle distribution after rotation was generated by simulating the RF manipulations in the PS without intensity effects. Bunches were matched at PS flat top before rotation with the distribution defined by Eq. (2.108). The nominal PS RF program for bunch rotation was used. In simulations, the 12 rotated bunches were injected into the SPS single and double RF systems ($V_{200}/V_{800} = 0.1$) and the results are compared in Fig. 4.18 with measurements done in a single RF system configuration. The measured stability threshold in the single RF system is reproduced in simulations when the particle distribution produced by the bunch rotation is used. Simulations of the same beam as in the single RF case but in the double RF system with a voltage ratio of 0.1 does not show improvement of beam stability, the instability threshold is actually even reduced.

For the bunch generation in the PS, different values of μ in the binomial distribution from Eq. (2.108) have been used. Figure 4.18 presents the results with $\mu = 1$, but similar stability limits are obtained for larger values of μ up to 2 for the same FWHM bunch length. Larger values of μ have also been studied and the stability threshold decreases significantly.

If flat bottom instabilities are cured by the OTFB and the FF systems, the 800 MHz RF system becomes more efficient and the voltage ratio between the two RF systems should be kept at 0.1 to improve the beam stability at flat bottom, as shown in Fig. 4.16 (left).



Figure 4.18 – Measured and simulated instability thresholds at flat bottom as a function of the bunch length after filamentation for a batch of 12 bunches. In simulations, the particle distribution is generated from the bunch rotation in the PS. The simulated thresholds in a single RF system are compared with the beam measurements under the same conditions where the OTFB and the FF systems are off and a 200 MHz voltage of 4.5 MV. The threshold simulated in the double RF operation (r = 0.1) is also shown.

4.4 Effect of 800 MHz RF System on Beam Stability at the SPS Flat Top Energy

Contrary to flat bottom, for LHC bunches at the SPS flat top, the plateau in the synchrotron frequency distribution is not inside the bunch with a nominal emittance at injection (0.35 eVs). Significant improvement of the beam stability can therefore be obtained by increasing the voltage ratio between the two RF systems. Figure 4.19 shows the maximum amplitude of the bunch length oscillations at flat top of 12 bunches normalised by the average bunch length at the end of the cycle as a function of the injected bunch intensity. The cases $V_{800}/V_{200} = 0.10$, 0.15, 0.20, and 0.25 are presented. In the case of a nominal voltage ratio (0.1), large oscillations are observed at high bunch intensity ($N_b \sim 2.3 \times 10^{11}$ ppb) which are suppressed when the voltage ratio is increased ($r \ge 0.20$). The instability for a batch containing 12 bunches can be cured at flat top by increasing the voltage ratio between the two RF systems to 0.25. The increase of stability for larger voltage ratio (0.15) was also confirmed during the SPS operation for batches of 48 bunches. Since the measured stability threshold at flat top and flat bottom is well reproduced in simulations with 12 bunches (case without OTFB, see Fig. 4.7 and Fig. 4.18), simulations are then used to study the effect of the 800 MHz RF system on beam stability with a nominal batch containing 72 bunches.

In the SPS, the stability threshold has a minimum value at flat top [19]. The simulations



Figure 4.19 – Measured maximum relative amplitude of the bunch length oscillations of 12 bunches at flat top as a function of the average injected bunch intensity. The injected longitudinal bunch emittance was nominal (0.35 eVs). The cases $V_{800}/V_{200} = 0.10$ (blue), 0.15 (green), 0.20 (grey), and 0.25 (red) are presented. The 200 MHz voltage at flat top was 7.0 MV and the OTFB and the FF were on.

were done at a constant momentum of 450 GeV/c. Bunches at flat top were assumed to be matched to the RF bucket with intensity effects. A batch of 72 bunches spaced by 25 ns was generated with particle distribution described by the binomial function in Eq. (2.108). The parameter μ of the distribution was fixed to the value 1.5 to fit the measured bunch profiles. The bunch length computed in simulations, like in measurements, through the FWHM of the bunch profile, was rescaled to 4σ , assuming a Gaussian distribution, see Eq. (2.71). The bunch emittance and intensity were varied in simulations to obtain the stability map with the present SPS longitudinal impedance model (Fig. 3.7). The fundamental impedance of the 200 MHz RF system was reduced by the OTFB and the FF systems by -20 dB. The maximum 200 MHz voltage was fixed to 7 MV and the possible power limitation was neglected since the goal of studies was to observe the effect of the fourth harmonic RF system for future intensities. This power limitation will be raised in the future, see Section 3.1. The simulated time at flat top is two seconds (compared to the 500 ms in the SPS operation) to observe slowly growing instabilities. In relevant intensity range, up to 2.5×10^{11} ppb, the multi-bunch instabilities are usually violent and appear before 500 ms. The maximum voltage ratio is fixed by the ratio of the harmonic numbers $h_{200}/h_{800} = 0.25$ since for larger values higher harmonics buckets appear inside the 200 MHz bucket. Therefore, in simulations we varied the voltage ratio in the



Figure 4.20 – Stability thresholds simulated for 72 bunches at flat top as a function of the average bunch length for different voltage ratios with $V_{200} = 7$ MV and $V_{800} = r \times V_{200}$. The present SPS longitudinal impedance model is used. A reference beam measurement with four batches of 72 bunches, with nominal emittance at injection (0.35 eVs), from Ref. [7] is also shown as a dot. The reference measurement was done at flat top in a double RF system with $V_{200} = 7.0$ MV and r = 0.1. The maximum amplitude of the bunch length oscillations during cycle (normalized to the average) exceeding 7% was used as a criterion to separate stable beams from unstable in simulations.

range (0.1, 0.25). After the RF upgrade, a maximum ratio of 0.16 will be achievable for HL-LHC intensity since the 200 MHz voltage at flat top will be increased to 10 MV, see Ref. [79].

The stability thresholds for the different voltage ratios are shown in Fig. 4.20. A beam measurement of the stability limit for an average bunch length of 1.65 ns, used for reference for the nominal LHC beam, is also indicated and agrees well with simulations [7]. For HL-LHC, the SPS performance will be pushed to its limits. Increasing the voltage ratio at flat top up to 0.25 increases the stability threshold. With the largest value, the intensity limit is doubled for the nominal bunch length of 1.65 ns. Simulations for the situation after LIU upgrades also show that an increase in the voltage ratio can improve the stability even beyond the scope of the HL-LHC project, see Chapter 5. However, other limitations should be taken into account, one of them is the beam loading in the 200 MHz RF system.

Optimization of the voltage ratio during the whole cycle taking into account various limitations is presented in the next section.

4.5 Optimization of the 800 MHz Voltage During the Cycle

We are interested in the maximum emittance, ϵ_{max} , a bunch can have before containing a flat portion of the synchrotron frequency distribution. To determine an optimal program for the voltage ratio during the acceleration cycle we define the following critical emittance

$$\epsilon_c = \min\{0 < \tilde{\epsilon} \le A \text{ such that } \omega'_s(\tilde{\epsilon}) = 0\},\tag{4.2}$$

where *A* is the bucket area (acceptance). This variable indicates for which value of its emittance a bunch contains a plateau in the synchrotron frequency distribution where the derivative with respect to the emittance goes to zero. In this case $\epsilon_{max} = \epsilon_c$, for longitudinal emittances larger than ϵ_c the Landau damping can be lost depending on intensity. If the synchrotron frequency distribution does not have a plateau, ϵ_c does not exist and $\epsilon_{max} = A$.

The synchrotron frequency and its derivative are computed numerically during cycle without intensity effects. The evolution of ϵ_{max} during cycle is shown for different voltage ratios in Fig. 4.21. In the cases $r \le 0.2$, the derivative of the synchrotron frequency distribution does not vanish during the cycle but it becomes very close to zero at flat bottom for r = 0.2.

The intensity effects (potential well distortion) also modify the synchrotron frequency distribution and may result in a loss of Landau damping. Taking into account the full bucket after filamentation, a voltage ratio r = 0.1 is more favourable for beam stability at flat bottom.

As one can see from Fig. 4.21, during acceleration (cycle time from 11.1 s to 19.5 s), the voltage ratio can be gradually increased to reach the value of 0.25 at flat top. The resulting voltage program is plotted in Fig. 4.22. These settings have been tested under real conditions with up to four batches of 12 bunches and improvement of beam stability was demonstrated for two SPS optics (Q20 and Q22) with a bunch intensity up to 2.3×10^{11} ppb. The evolution of the



Figure 4.21 – Maximum longitudinal bunch emittance ϵ_{max} during cycle for different voltage ratios *r*; it is the critical emittance defined by Eq. (4.2) if it exists or the bucket area *A* otherwise.



Figure 4.22 – Optimized voltage ratio *r* between the two SPS RF systems during acceleration cycle for the LHC proton beam (the 200 MHz voltage from Fig. 3.4).

bunch length during the cycle with the nominal voltage ratio (0.1) and the optimized program in the Q20 optics is shown in Fig. 4.23 for comparison. The average bunch length of the third batch (from four) along the cycle is presented. In the case of the optimized program, bunches are less affected by the uncontrolled longitudinal emittance blow-up during acceleration and



Figure 4.23 – Evolution of the bunch length during the SPS cycle for 12 bunches with a bunch intensity $N_b = 2.3 \times 10^{11}$ ppb, in the Q20 optic. The nominal case (solid lines) is compared with the optimized voltage ratio program (dashed lines). The two 800 MHz voltage programs are shown and the 200 MHz voltage program is the LHC cycle from Fig. 3.4, and the OTFB and the FF systems are on.

the final bunch length is smaller, which demonstrates the improvement of beam stability. However, one should also remember that intensity effects modify the synchrotron frequency distribution. In simulations for high intensity $(2.3 \times 10^{11} \text{ ppb})$ beam, the synchrotron frequency distribution is affected by the induced voltage differently for each bunch. The synchrotron frequency in the bunch centre is reduced by 4% for the first bunch and by 11% for the 12th bunch. As a next step in the optimization, the collective effects could be considered in the design of the voltage program.

4.6 Conclusion

The longitudinal multi-bunch instabilities in the SPS have been studied by means of beam measurements compared to analytical estimations and particle tracking simulations. In a single RF system, we established that a coupled-bunch instability, at the end of the acceleration ramp and at flat top, is likely to be caused by the 630 MHz HOM of the 200 MHz RF system. Analytic estimations of the corresponding threshold agree in the last part of the acceleration cycle with measurements with beams of nominal emittance when the OTFB and the FF systems are activated and deactivated. This is also the case when the emittance is smaller (0.25 eVs). Together with the stability threshold shown in Section 3.3.1, this is another indication of the importance of the 630 MHz HOM in the formation of the instability at flat top and, here, also

during the last part of the acceleration ramp.

The effect of the 200 MHz LLRF systems was also studied in a single RF system during the acceleration ramp. The OTFB and the FF systems act together to reduce the fundamental impedance of the 200 MHz RF system. The measured stability thresholds with the OTFB but without the FF do not exhibit a significant difference from the stability threshold measured when both are activated. When the OTFB and the FF systems are both activated, no instability was developing at flat bottom for the beams studied. The beam measurements with different LLRF settings allowed to separate instabilities appearing at flat bottom and the beginning of the ramp, which are certainly due to the fundamental impedance of the 200 MHz system, from the instabilities at the end of the ramp and at flat top, which are caused by the impedance of the 630 MHz HOM. The instability at flat bottom is cured by the OTFB and the FF systems in the bunch intensity range of the beam measurements (12 bunches with an intensity $N_b < 2.5 \times 10^{11}$ ppb).

We also revealed that, in our sets of measurements above a bunch intensity of 1.4×10^{11} ppb, the injected 12 bunches have a larger longitudinal emittance which increases with the bunch intensity. This explains the absence of energy dependence of the stability threshold during the first part of the acceleration cycle, that was observed in measurements. However, in standard operations, the PS provides a constant longitudinal emittance. Particle tracking simulations confirm the modifications of the stability threshold with a larger bunch length after filamentation. This intensity dependence should be cured in the PS to avoid larger bunches in the SPS which would increase the amount of beam losses. Another possibility would be to inject the larger bunches in a lower harmonic RF system in the SPS, scenario presented in Chapter 6.

We showed for the double RF operation that the effect of the second RF system on the beam stability is different at flat bottom and at flat top due to the position of the plateau in the synchrotron frequency distribution within the bunches. At flat bottom the synchrotron frequency derivative goes to zero within the nominal longitudinal emittance (0.35 eVs) and the beam becomes more unstable when the ratio between the two RF systems is increased above the nominal value of 0.1. This value should be conserved during the whole flat bottom to guarantee beam stability. However, at flat top the synchrotron frequency derivative does not go to zero within the longitudinal bunch emittance (0.35–0.5 eVs) and beam stability is improved when the voltage ratio between the two RF systems is increased. A maximum ratio of 0.25 can be reached at flat top in theory. Trying to avoid the plateau of the synchrotron frequency distribution to be within the bunches during the acceleration, we also computed an optimized cycle for the voltage ratio between the two RF systems which was successfully implemented, tested in operation and showed an improvement of beam stability. This stable beam of 12 bunches was injected for the first time into the LHC to probe in various studies high bunch intensities.

In this chapter it was also demonstrated again that particle tracking simulations are able to

reproduce the measured thresholds using the present longitudinal impedance model of the SPS and the correct particle distribution (bunches rotated in the PS). Predictions of beam stability after LS2 are done with this model modified to include LIU upgrades and this is the subject of the next chapter.

5 Longitudinal Beam Stability in the SPS after LIU Upgrades

Intensity limitations presented in Chapter 3 and specifically multi-bunch instabilities studied in detail in Chapter 4, must be overcome to allow the production of the HL-LHC beam containing four batches of 72 bunches spaced by 25 ns with a bunch intensity of 2.3×10^{11} ppb.

Due to longitudinal instabilities and potential well distortion, the 200 MHz voltage necessary to ensure the stability of the HL-LHC beam at SPS flat top with 1.65 ns bunch length was estimated to be around 12 MV [80]. However, the maximum voltage available at HL-LHC intensity (see Fig. 3.3) after the RF upgrade will only be 10 MV. This is limited by the RF power available in the SPS (see Fig. 3.3) but also in the LHC, where too large bunch emittances will require more 400 MHz voltage. The RF voltage of 10 MV in the SPS corresponds to a matched longitudinal bunch emittance of 0.57 eVs which is larger than the 0.35 eVs at injection, but this increase of emittance by controlled blow-up would not be sufficient to ensure the beam stability with the present longitudinal impedance of the SPS ring, as shown in Section 5.1, and additional measures are needed. The two possible ways for improving beam stability in the SPS are to use the 800 MHz RF system in bunch-shortening mode with larger voltage ratio, as discussed in the previous Chapter and, in addition, to reduce the longitudinal impedance.

In this chapter, the impact on beam stability of baseline LIU upgrades [3] is analysed. Possible ways of improving further beam stability are also studied using particle tracking simulations and analytical estimations. All simulations presented in this chapter are done for a double RF system. The voltage ratio between the two RF systems of 0.1 is used as a basis to compare the improvement of beam stability with larger values. The bunch distribution in the longitudinal phase space used in simulations is the binomial function defined by Eq. (2.108) with $\mu = 1.5$, which corresponds to bunch profiles measured at flat top in the present configuration.

The first section investigates the effect of the LIU RF upgrades on the stability thresholds at SPS flat top. The impact on beam stability of the different HOMs of the 200 MHz RF system is studied in Section 5.2. In the third section, the longitudinal impedance reduction of the SPS ring, which is in the baseline of the LIU project [3], is presented, and possible increase of the stability threshold is calculated. The effect on beam stability of further possible reductions

of the longitudinal impedance of the SPS is investigated in Section 5.4. Section 5.5 explores possible increases of the stability threshold after LIU upgrades by increasing the voltage ratio between the two RF systems and the last section presents the effect on the stability threshold of the voltage limitation due to beam loading.

5.1 The SPS 200 MHz RF System Upgrade

The beam loading in the main RF system of the SPS may have a negative impact on the beam quality, see Section 3.1. The voltage induced by the beam at the fundamental frequency of the 200 MHz RF system limits the voltage available for the beam, due to the limited RF power, and the amplitude of the induced voltage increases with the cavity length square, as it appears in the expression of the fundamental impedance of the cavity in Eq. (3.3). The upgrade of the 200 MHz RF system aims at reducing the impedance of the TW cavities seen by the beam. Figure 5.1 (left) shows the maximum voltage that a 200 MHz TW cavity can provide to the beam as a function of the cavity length at HL-LHC intensity. Two cases are presented, where the peak RF power available is respectively P = 1.6 MW and P = 1.05 MW. These values will be possible after the RF power plants upgrade in pulsing mode (zero voltage without beam). The LLRF system will also be upgraded to cope with the pulsing mode. As one can see from Fig. 5.1 (left), at the HL-LHC intensity (2.3×10^{11} ppb) for the same RF input power, the five-section TW cavity becomes less efficient than shorter cavities at providing RF voltage.

Several options were possible to rearrange the existing four cavities (with two spare sections) into five or six shorter cavities. The option that has been selected is to rearrange the two five-section TW cavities using the two spare sections to form four three-section cavities [11].



Figure 5.1 – Left: maximum voltage at the HL-LHC intensity $(2.3 \times 10^{11} \text{ ppb})$ as a function of the 200 MHz TW cavity length for two values of the RF input powers of 1.6 MW (blue) and 1.05 MW (orange). Right: maximum emittance that can be accelerated after LIU RF upgrades as a function of the filling factor q_p , constant during the nominal LHC cycle in the Q20 optics, where the RF voltage program is computed iteratively for each emittance and filling factor like in Fig. 3.5.

The existing four RF power plants will deliver to each of the three-section cavity a peak power of 1.05 MW (in pulsing mode). The present two four-section cavities will remain but two new power plants will be added to deliver a peak power of 1.6 MW (in pulsing mode) to compensate for the increase of the beam loading for higher beam intensities. The RF parameters for the three-, four-, and five-section TW cavities of the 200 MHz RF system are given in Table 3.1 and are reminded here in Tab. 5.1.

Table 5.1 – RF parameters of the 200 MHz travelling-wave structures in the SPS. The values of the average and peak power for the three- and four-section cavities are the one after the RF upgrade, the five-section cavities will disappear after LS2.

	200 MHz			
	5-section	4-section	3-section	
Interaction length L [m]	20.196	16.08	11.97	
Series impedance $R_2 [k\Omega/m^2]$	27.1			
Filling time L/v_g [µs]	0.712	0.568	0.422	
Beam loading impedance $L^2 R_2/8 [M\Omega]$	1.381	0.879	0.485	
Power (average/ peak) [MW]	0.75/1.0	0.75/1.6	0.75/1.05	

The maximum total 200 MHz voltage available at HL-LHC intensity $(2.3 \times 10^{11} \text{ ppb})$ is 10 MV, as explained already in Section 3.1. This value, $V_{200} = 10$ MV, is used in simulations for the evaluation of the stability thresholds at flat top after LIU upgrades. Simulations including the voltage reduction from beam loading are presented in Section 5.6. The 800 MHz RF system was already upgraded during the long shutdown 1 (2013–2014) and the maximum 800 MHz voltage at HL-LHC intensity is 1.6 MV, which could be pushed to 1.7 MV in pulsing mode [79]. Then, the maximum voltage ratio between the two RF systems after RF upgrades will be $V_{800}/V_{200} = 0.16$ for $V_{200} = 10$ MV. Larger ratios can be used for smaller 200 MHz voltages (and smaller longitudinal emittance).

The maximum emittance that can be accelerated after RF upgrade is computed like in Section 3.1, and is shown in Fig. 5.1 (right) as a function of the filling factor q_p .

The nominal longitudinal bunch emittance at injection is 0.35 eVs and controlled emittance blow-up should be applied during the acceleration ramp. At flat top, with $V_{200} = 10$ MV, the bucket area is 2.8 eVs and the matched longitudinal bunch emittance for a bunch length of 1.65 ns is $\epsilon = 0.57$ eVs, computed using Eq. (2.70), which corresponds to $q_p \approx 0.5$. Regarding the fundamental impedance of the 200 MHz RF system, the shortening of cavities will reduce the total shunt impedance by 18%. The real and imaginary parts of the fundamental impedance of the present and future RF systems are compared in Fig. 5.2, using the G. Dome's formula in Eq. (3.2). Moreover, the upgraded OTFB and FF systems will decrease this value by at least -26 dB [81], to be compared to the present -20 dB. The improvement of the LLRF will be implemented during LS2 and it is not possible yet to have a model of the OTFB and the FF to be used in simulations. The assumption of a constant reduction by -26 dB in the 200 MHz passband is used in the simulations presented in this chapter.



Figure 5.2 – Beam-coupling impedance of the 200 MHz TW cavities for the cases before (blue) and after (orange) LIU RF upgrade. The solid lines correspond to the real part and the dashed lines to the imaginary part. The effect of the OTFB and the FF systems are not included and would reduce the impedance by -20 dB before upgrades and by -26 dB after upgrades.



Figure 5.3 – Instability thresholds at flat top simulated for 72 bunches in a double RF system before (black) and after (orange) the RF upgrade. The 200 MHz voltage after upgrade is 10 MV and 7 MV before, the voltage ratio between the two RF systems $V_{800} / V_{200} = 0.1$. The SPS longitudinal impedance model, presented in Section 3.2, is used with the impedance at 200 MHz reduced by -18% (shortening of cavities) and then by -26 dB (OTFB and FF systems) in the case after the RF upgrade.

The stability threshold at flat top for 72 bunches spaced by 25 ns in a double RF system after the RF upgrade (10 MV) is shown in Fig. 5.3 together with the stability threshold before the upgrade (7 MV). The measured bunch length spread of $\pm 10\%$ along the batch [7] is indicated at HL-LHC intensity. The average bunch length is 1.65 ns but a maximum spread of $\pm 10\%$ was observed in beam measurements at lower intensities. Therefore, the stability threshold at HL-LHC intensity must have sufficient margins around 1.65 ns to ensure the beam stability. In simulations bunches with a constant emittance were used. The longitudinal impedance model presented in Section 3.2 is used with only the 200 MHz impedance modified. The higher longitudinal bunch emittance, due to the larger available voltage, corresponding to the same bunch length, increases significantly the stability threshold of the HL-LHC beam. The intensity limit at 1.65 ns is increased by 50% due to the RF upgrade but this is not sufficient to ensure beam stability at HL-LHC intensity (2.3×10^{11} ppb). Further upgrades, such as a reduction of the longitudinal SPS impedance, are necessary. The effect on beam stability of the impedance of the four HOM bands in the 200 MHz TW cavities, presented in Section 3.2, is studied in the next section for the situation after the RF upgrade.

5.2 Effect of the HOMs of the 200 MHz RF System On Beam Stability

The HOM bands of the SPS 200 MHz TW cavities are another important contribution to the SPS impedance budget. As it has been shown in Section 3.3, the impedance of the 630 MHz HOM can be responsible for the instabilities of the LHC beam. The impact on the HL-LHC beam stability of the four HOM bands was studied using particle tracking simulations. The stability thresholds were simulated by removing sequentially the impedance of each HOM from the longitudinal impedance model of the SPS and the results are shown in Fig. 5.4.

As can be seen in Fig. 5.4, from the four HOMs existing in the 200 MHz RF cavities of the SPS, only the impedance of the 630 MHz HOM has an impact on the beam stability but the coupled-bunch instability threshold in Eq. (2.112) predicts that the threshold from the impedance of the 630 MHz and the 915 MHz HOMs should be similar. Indeed, for a given bunch with length τ , the stability limit scales for the different HOMs like

$$N_b \sim \frac{G(f_r \tau)}{R_{\rm sh}},\tag{5.1}$$

where f_r and R_{sh} are the resonant frequency and the shunt impedance of the HOMs, given in Tab. 5.2. Figure 5.5 shows the *G* function defined in Eq. (2.112) as a function of the resonant frequency of the impedance (left) and the scaling of the threshold defined in Eq. (5.1) as a

Table 5.2 – Maximum shunt impedance, $R_{\rm sh}$, for the four HOM bands of the 200 MHz TW structures.

Resonant frequency f_r [MHz]	630	915	1130	1500
Maximum shunt impedance R_{sh} [M Ω]	0.57	1.56	0.45	0.60



Figure 5.4 – Instability thresholds at flat top simulated with 72 bunches in the double RF system with $V_{200} = 10$ MV and $V_{800}/V_{200} = 0.1$. The present longitudinal SPS impedance model is used (as presented in Section 3.2). The threshold after RF upgrades (solid orange line) is compared with the cases where the impedance of the four HOMs of the 200 MHz RF cavities are sequentially removed from the full SPS longitudinal impedance model (vacuum flanges are not shielded).



Figure 5.5 – Left: G function from the analytic expression of the coupled-bunch stability threshold in Eq. (2.112) for a bunch length of 1.65 ns (flat top) as a function of the resonant frequency f_r . The vertical lines indicate the resonant frequencies of the impedance of the four HOM bands of the 200 MHz RF system. Right: Scaling of the stability threshold defined in Eq. (5.1) as a function of the bunch length in the SPS for the four HOMs of the 200 MHz RF system at 630 MHz (blue), 915 MHz (orange), 1130 MHz (green) and 1500 MHz (red).

function of the bunch length for the four HOMs (right). At flat top, for a bunch length of 1.65 ns, the thresholds corresponding to the 630 MHz and the 915 MHz HOM should be similar according to Eq. (5.1). The thresholds for the two other HOMs are much higher due to their higher frequency and their lower shunt impedance. Results of simulations in a single and a double RF system using only the impedance of the 630 MHz and the 915 MHz are presented in Fig. 5.6. The thresholds are similar at a bunch length of 1.65 ns, as predicted. The difference in the threshold increases in a double RF system with $V_{800} / V_{200} = 0.1$ and the threshold of the 630 MHz HOM is the lowest. Nevertheless, simulations using the full longitudinal impedance model of the SPS before and after LIU upgrades do not exhibit an impact of the 915 MHz on the stability threshold of a single batch. However, the value of Q for the 915 MHz HOM between 3000 and 5000 give an e-folding time between 1000 µs and 1800 µs, which corresponds to the length of one batch of 72 bunches. Therefore the SPS batches with 200 ns gaps are coupled through their respective wakefields and the effect of this HOM on the beam spectrum when the ring is full was already observed in Ref [47]. Indeed, using the definition of the wakefield in Eq. (2.60), the maximum value of the wakefield accumulated along the beam, Σ , of M bunches spaced by τ_{bb} can be written

$$\Sigma = 2aR_{\rm sh} \sum_{n=0}^{M} e^{-an\tau_{bb}} = 2aR_{\rm sh} \frac{1 - e^{-a(M+1)\tau_{bb}}}{1 - e^{-a\tau_{bb}}},$$
(5.2)

where a was defined in Eq. (2.60). Assuming a long range wakefield (large Q) such that



Figure 5.6 – Stability thresholds for 72 bunches in single (dashed) and double (solid) RF system at flat top with the impedance of the 630 MHz HOM (blue) or the 915 MHz HOM (orange). The 200 MHz voltage is $V_{200} = 10$ MV and $V_{800}/V_{200} = 0.1$.

 $a(M+1)\tau_{bb} \ll 1$, Eq. (5.2) can be approximated by

$$\Sigma \approx 2aR_{\rm sh}(M+1) = \omega_r \frac{R_{\rm sh}}{Q}(M+1).$$
(5.3)

The value of $R_{\rm sh}/Q$ is 2850 for the total impedance of the 630 MHz HOM and 300 for the 915 MHz HOM, almost 10 times smaller. This difference in $R_{\rm sh}/Q$ makes the effect of the 630 MHz HOM more significant for a small number of bunches, as it is the case for a single batch. When the number of bunches increases (multi-batch), the approximation (5.3) is not valid but the value of Eq. (5.2) can be computed numerically. The ratio $\Sigma_{915}/\Sigma_{630}$ is shown in Fig. 5.7, where Σ_{915} and Σ_{630} are defined in Eq. (5.2) and computed using the parameters of the 915 MHz HOM and the 630 MHz HOM, respectively. The 915 MHz HOM becomes more significant compared to the 630 MHz HOM for multi-batch beams. This result was confirmed in simulations, as explained below.

The very recent development of the MPI (Message Passing Interface) version of the simulation code BLonD allowed to study the stability of the four SPS batches regarding the 915 MHz HOM [82]. For an initial $4-\sigma$ bunch length of 1.60 ns an intensity of 2.90×10^{11} ppb, above the stability threshold as it will be shown in Section 5.3, the bunch length along the four batches (spaced by 200 ns) after 2.3 s, simulated at flat top in a double RF system in the situation after LIU upgrades is shown in Fig. 5.8 (left). In this case, it appears that the second batch becomes unstable earlier (in terms of the number of bunches along the batch) and all the bunches in the third and fourth batch are unstable. When the impedance of the 915 MHz HOM is removed from the model, the coupling between the batches disappears, as shown in Fig. 5.8 (right); each batch behaves similarly, independent of the preceding batches. To ensure the stability of the HL-LHC beam, the impedance of this HOM must be reduced. Simulations of four batches



Figure 5.7 – Ratio of the sum defined in Eq. (5.2) for the 630 MHz HOM and the 915 MHz HOM. The vertical dashed lines indicate the position of the batch heads with 200 ns gaps. In reality the wakefield decreases in this gap, which is not taken into account in the calculation.



Figure 5.8 – The 4- σ bunch length (at 2.3 s) along the four SPS batches simulated at the SPS flat top energy in a double RF system with a 200 MHz voltage of 10 MV and a voltage ratio between the two RF systems of 0.1 for a bunch intensity of 2.90×10^{11} ppb and an initial bunch length of 1.60 ns in the case of the SPS impedance model after LIU upgrades (Fig. 5.13) (left) and where the 915 MHz HOM has been removed (right). The vertical dashed lines indicate the separation (200 ns) between the batches.



Figure 5.9 – The 4- σ bunch length (at 2.3 s) along the four SPS batches simulated at the SPS flat top energy in a double RF system with a 200 MHz voltage of 10 MV and a voltage ratio between the two RF systems of 0.1 for a bunch intensity of 2.44×10^{11} ppb and an initial bunch length of 1.60 ns in the case of the SPS impedance model after LIU upgrades (Fig. 5.13) (left) and where the 915 MHz HOM has been damped by a factor 2 (right). The vertical dashed lines indicate the separation (200 ns) between the batches.

of 72 bunches with a gap between batches of 200 ns using a bunch intensity of 2.44×10^{11} ppb, below the stability threshold, but very close (see Section 5.3), have been carried out to identify the necessary impedance reduction. Results are shown in Fig. 5.9 for the situation after LIU upgrades (left) and the case where the impedance of the 915 MHz HOM was damped by a factor 2 (right). The first batch (72 bunches) is stable in both cases since the bunch intensity is below the stability threshold for one batch, but when the 915 MHz HOM is not damped the third and fourth batch are unstable. However, when the impedance of this HOM is damped by a factor 2, the four batches are stable.

Chapter 5. Longitudinal Beam Stability in the SPS after LIU Upgrades

The effect of the 915 MHz HOM on the multi-batch stability is still under investigation but the latest simulations at flat top energy, using a refined model of the impedance of the 915 MHz HOM (not used in this thesis), suggest that the instability threshold of four batches of 72 bunches could actually be rather very similar to the single batch case. The results obtained for a single batch may be pertinent in the multi-batch case even without damping of the 915 MHz HOM and we assume below that the 915 MHz HOM will be sufficiently damped if needed.

The impedance of the 630 MHz HOM must also be reduced to reach HL-LHC intensity in the SPS. However, as mentioned in Section 3.2, this impedance is already heavily damped by a series of RF couplers and further significant impedance reduction is complicated to achieve.

The baseline scenario of impedance reduction to reach the HL-LHC target and ways of increasing further the stability threshold by impedance reduction of different contributions, including the 630 MHz HOM are presented in the next two sections.

5.3 Baseline Impedance Reduction of the SPS

In Section 3.3 it was shown that the impedance of the HOM at 630 MHz and the QF flanges give the lowest stability threshold for 72 bunches at flat top in the present situation (7 MV). The impedance of these two elements will be reduced during LS2, in addition to the -26 dB reduction of the impedance at 200 MHz from the OTFB and the FF systems. Smaller modifications to the longitudinal impedance are also modelled but they are not considered here.

In this section the impedance model after upgrades, which are considered the baseline of the



Figure 5.10 – Left: CAD model of the impedance shield of the QF flanges using movable RF fingers [83] (yellow). Right: Total impedance of the QF type of vacuum flanges before shielding (red) and after the implementation of the impedance reduction (blue) [55].

LIU project, is presented. The impedance of the QF type of vacuum flanges around 1.4 GHz will be significantly reduced during LS2. A shield, which contains movable RF fingers as shown in Fig. 5.10 (left), will be installed on all QF flanges [55] in the short straight sections of the SPS ring. The shielded impedance is compared to the present impedance in Fig. 5.10 (right), and the impedance of the vacuum flanges responsible for the beam instability as shown in Section 3.3.1 is significantly reduced. These shields are now under implementation.

For future beam stability, the impedance of the HOM of the 200 MHz RF cavities at 630 MHz needs to be reduced. However, the impedance of the 630 MHz passband is already heavily damped by a series of RF couplers. Further impedance reduction was difficult to achieve. The impedance of the mode in the present situation is shown in Fig. 5.11, compared to the case where additional damping by a factor of three is assumed, keeping the ratio $R_{\rm sh}/Q$ of the mode constant. The factor 3 is the impedance reduction that should be attained to ensure the beam stability of the HL-LHC beam. This requirement was obtained in particle tracking simulations at flat top assuming different values for impedance reduction and the stability thresholds simulated for 72 bunches are shown in Fig. 5.12. In simulations, the longitudinal impedance model of the SPS after LIU upgrades, shown in Fig. 5.13, was used. The cases where the HOM is not damped or removed from the model are also presented. The damping by a factor of three, assumed in the baseline impedance reduction, is desired to guarantee stability of the HL-LHC beam including bunch length spread.

Finally, the longitudinal impedance model of the SPS after LIU upgrades is presented in Fig. 5.13.



Figure 5.11 – Total impedance of the HOM at 630 MHz in the 200 MHz RF cavities now (fiveand four-sections) (red), and after RF upgrades (three- and four-sections) and damping by a factor 3 assuming a ratio $R_{\rm sh}/Q$ constant (blue).



Figure 5.12 – Stability threshold of 72 bunches simulated at flat top in a double RF system with a 200 MHz voltage of 10 MV and a voltage ratio between the two RF systems V_{800}/V_{200} of 0.1. The longitudinal impedance model after LIU upgrades, including the shields of the QF, is used. Different values for the damping of the 630 MHz HOM are relative to the present (already damped) case.



Figure 5.13 – Longitudinal impedance model (|Z|) of the SPS before (red) and after (blue) LIU upgrades. The reduction of 200 MHz impedance from the RF upgrade and the LLRF system upgrade are included with the 630 MHz HOM damping by a factor 3 and the shielding of the QF flanges [3, 8, 54–56, 58].

The parameters of the different resonators are listed in Appendix B. The impedance reduction due to the RF upgrade, the LLRF system upgrade, the HOM damping, and the shielding of vac-



Figure 5.14 – Stability threshold of 72 bunches simulated at the flat top energy. The threshold in the present situation (black) is compared with the case after RF upgrades only (i.e. power and LLRF) (orange) and the situation after all LIU upgrades (i.e. modelling also the impedance reduction) (blue). Simulations are done in a double RF system with a 200 MHz voltage of 10 MV and a voltage ratio between the two RF systems of 0.1.

uum equipment are included. The model corresponds to the baseline of the LIU impedance reduction campaign. The stability thresholds simulated for the (HL-)LHC batch at flat top are presented in Fig. 5.14. The stability threshold after LIU upgrades covers the target of the HL-LHC beam. However, it does not give sufficient margins of beam stability to account for the operational spread of bunch length (\sim 10%) and to the possible losses at SPS-LHC transfer, such that higher bunch intensity before extraction to the LHC is needed. Different ways of enhancing beam stability are presented in what follows. Different possibilities of impedance reduction that could give margins for beam stability at HL-LHC intensity are discussed in the next section.

5.4 Options for Further Impedance Reduction

5.4.1 MKP Shielding

The SPS ring contains 16 unshielded kickers for proton beam injection called MKP. Their impedance is of a broadband type and can have a significant impact on the loss of Landau damping and the single-bunch stability. It is possible to apply serigraphy on these kickers and reduce their impedance to a level where it becomes negligible in comparison to the



Figure 5.15 – Stability thresholds of 72 bunches simulated at flat top in a double RF system with the 200 MHz voltage of 10 MV and $V_{800}/V_{200} = 0.1$. The case baseline LIU upgrades (blue) is compared to the situation where serigraphy has been applied to the MKP kickers (green).

longitudinal impedance model [84]. In this case, the stability threshold of the LHC beam could be increased. Results of simulations using the SPS longitudinal impedance model after LIU upgrades with removed impedance of all MKP kickers are shown in Fig. 5.15. In the range of bunch lengths of interest, the intensity limit is increased by 13%. The shielding of the MKP kickers allows to significantly increase the margins for the stability of the HL-LHC beam, and can be implemented in the future, if necessary.

5.4.2 Shielding of Sector Valves

In the SPS, 64 sector values are installed which give the possibility to close different parts of the machine, in case of intervention, and to conserve the vacuum [61]. There exists a shielded version of the same values that could be used. It has not been done in the first place due to budget considerations. The effect of this shield on beam stability, studied in simulations with 72 bunches at flat top, is shown in Fig. 5.16. Installing the new values with shielding could give the additional safety margin for the stability threshold to reach the HL-LHC intensity.

5.4.3 Another Way of Reducing the 630 MHz HOM Longitudinal Impedance

The HOMs in the 630 MHz passband are critical for beam stability and further damping is difficult to achieve. Other ways of reducing the total impedance seen by the beam were



Figure 5.16 – Stability threshold of 72 bunches simulated at flat top in a double RF system with a 200 MHz voltage of 10 MV and $V_{800}/V_{200} = 0.1$. The case of the longitudinal impedance model after LIU upgrades (blue) is compared to the model where the 64 sector vacuum valves have been replaced by new shielded valves.

investigated. The natural spread of the HOMs between cavities has been measured to be around 100 kHz [58]. The total shunt impedance seen by the beam is not sufficiently reduced by this spread and detuning the mode differently for each cavity, mechanically or by using new RF couplers, has been considered as a solution to improve stability. Particle simulations show that a frequency spread of the HOM from cavity to cavity by a few MHz could already significantly improve the situation. Moreover, a shift of the mode by 10 MHz gives an increase of the intensity threshold up to 50%. However, the mode in the 630 MHz passband appears to be rigid. Most of the stored energy is in the cavity volume [57] and the frequency cannot be shifted by 10 MHz by known RF couplers. To understand the possible gain of a smaller shift, the mechanism behind the improvement of beam stability was studied and is discussed below in more detail.

The total impedance of the HOMs of the four three-section cavities and two four-section cavities without further damping can be modelled by a resonator at $f_r = 630$ MHz with a shunt impedance $R_{\rm sh} = 570$ k Ω and a quality factor Q = 200, see Eq. (2.59). The longitudinal impedance model after LIU upgrades in Fig. 5.13 was used in simulations done in a double RF system with $V_{200} = 10$ MV and $V_{800}/V_{200} = 0.1$. Figure 5.17 shows the stability thresholds for five different values of the resonant frequency between 620 MHz and 640 MHz. Shifting the original 630 MHz frequency by a few MHz can improve significantly the stability threshold. The HOM currently is in an asymmetric region of reduced beam stability. A frequency shift in the



Figure 5.17 – Stability thresholds for 72 bunches simulated at flat top in a double RF system with 10 MV at 200 MHz and 1 MV for 800 MHz RF system in bunch-shortening mode. The full SPS impedance model after LIU upgrades is used and the resonant frequency of the HOM in the 630 MHz passband is shifted by different values. The stability threshold corresponding to the baseline of impedance reduction (HOM damped by a factor of 3) is also shown (solid red line). The horizontal line indicates the HL-LHC beam intensity and the vertical one the nominal bunch length. This bunch length has a spread in present operation of \pm 10%, indicated by the shaded area.

positive direction increases stability further, different from a shift in the negative direction. For a resonant frequency of 640 MHz, simulations show a remarkable improvement of the stability threshold. This frequency corresponds to one of the beam spectrum lines (40 MHz) related to bunch spacing (25 ns). It is well known for a ring filled with equally spaced bunches, that a narrowband impedance with a frequency at a beam spectral line cannot drive instabilities [46], but the overlap of the beam spectrum and the impedance does increase power loss and heating. However, the 630 MHz HOM, already heavily damped to Q = 200, is not particularly narrowband and a train of 72 bunches occupies only 8% of the machine. The first bunch of the beam does not see the last bunch and beam spectrum lines are much broader than in the full ring case. Nevertheless, simulations exhibit a similar improvement of the stability threshold. Contrary to the ideal case of a full ring, the stability can be studied by solving the equations of motion from bunch to bunch [85]. Due to the complexity of the machine impedance, we choose to isolate the impedance of the 630 MHz and to simplify the equations by considering a single 200 MHz RF system. However, as shown later, simulations in a single and a double RF system demonstrate the same effect. The effect of a shift of the resonant frequency is studied below, using a single-particle model.

Stability of Train With Point-Like Bunches

A point-like bunch model is already able to explain observations made in simulations of the LHC beam. In this model, each of the *M* bunches is represented by a single rigid particle carrying the total bunch charge eN_b . The *i*-th bunch oscillates around its synchronous phase ϕ_i with a relative position in time $\tau_i = \frac{\phi_i}{\omega_{\text{RF}}}$, see Fig. 5.18. Initially centred in the RF bucket, bunches are separated by a distance τ_{bb} . The equations of motion are derived for small amplitudes of synchrotron oscillations without acceleration. This approximation is valid at the SPS flat top for a nominal bunch length of 1.65 ns. The behaviour of the *i*-th bunch is described by the equation

$$\ddot{\tau}_i + \omega_{s0}^2 \tau_i = \frac{\eta e}{\beta_s^2 E_s T_0} V_{\text{ind}}(\tau_i), \tag{5.4}$$

where V_{ind} is the voltage induced by the particles circulating ahead in the batch. The synchrotron frequency $f_{s0} = \omega_{s0}/2\pi$ in the bunch centre is defined by Eq. (2.26).

For a longitudinal line density $\lambda(\tau)$, the general expression of the induced voltage is given by Eq. (2.72). For point-like bunches, the line density takes only discrete values. The wake function can be expanded to linear order for small amplitude synchrotron oscillations. The zero-order term can be discarded and only the first-order term contributes to the dynamics. The equation of motion of the *i*th bunch, considering the interactions with all preceding neighbours, is

$$\ddot{\tau}_{i} + \omega_{s0}^{2} \tau_{i} = \mathcal{D} \sum_{k=0}^{i-1} W'[(i-k)\tau_{bb}](\tau_{i} - \tau_{k}),$$
(5.5)

with τ_{bb} = 25 ns for an LHC beam and

$$\mathscr{D} = \frac{\eta e^2 N_b}{\beta_s^2 E_s T_0}.$$
(5.6)

For a resonator with $Q \gg 1$, the derivative of the wake function from Eq. (2.60) can be approximated by

$$\frac{W'(\hat{\tau})}{2a^2R_{sh}} = -e^{-a\hat{\tau}}\sqrt{4 + \left(\frac{\bar{\omega}}{a} - \frac{a}{\bar{\omega}}\right)^2}\sin(\bar{\omega}\hat{\tau} + 1/Q),\tag{5.7}$$



Figure 5.18 – Schematic view of a train in the point-like bunch model.

where

$$a = \frac{\omega_r}{2Q}$$
 and $\bar{\omega} = \omega_r \sqrt{1 - \frac{1}{4Q^2}}$. (5.8)

The induced voltage cancels perfectly at every τ_{bb} position if the resonant frequency is

$$\omega_r = 2\pi \frac{k - \frac{1}{\pi Q}}{\tau_{bb} \sqrt{4 - 1/Q^2}}.$$
(5.9)

In the SPS case with a bunch spacing of 25 ns, this corresponds to frequencies $f_r \approx k \times 20$ MHz for k = 1, 2, 3.... If the resonant frequency of the mode is a multiple of 20 MHz, the growth rate of the instability is zero. For a bunch with a certain given particle distribution, the bunch centre sees an induced voltage close to zero. This explains the improvement of the beam stability for bunch trains for resonant frequencies of 620 MHz and 640 MHz observed in Fig. 5.17. From Eq. (5.5), the growth time of the instability can be computed for the *i*-th bunch taking into account all preceding bunches,

$$\frac{1}{\mathrm{Im}(\omega)_{i}} = \left[\mathrm{Im}\left(\omega_{s0}^{2} - \mathcal{D}\sum_{k=0}^{i-1} W'[(i-k)\tau_{bb}]\right)^{\frac{1}{2}}\right]^{-1}.$$
(5.10)

The expression (5.10) can be computed numerically for an intensity above the instability threshold considering different numbers of bunches coupled. The results are shown in Fig. 5.19. For the nearest neighbour interaction, the growth time appears to be symmetric



Figure 5.19 – Instability growth time calculated for a train of point-like bunches for an intensity above threshold. The sum in Eq. (5.10) is truncated for different lengths of interaction between bunches.

between the two 20 MHz lines. An asymmetry appears when the number of bunches coupled increases. After adding more than 10 bunches, the growth time function does not change significantly. This result is reasonable since the wake function decay time is over four bunches. However, compared to the realistic bunch case presented in Fig. 5.17, it suggests that a frequency shift in the positive direction degrades beam stability. This analytical estimation is confirmed by simulations of 72 point-like bunches, see Fig. 5.20.

Train of Bunches With Particle Distribution

When the particle distribution is taken into account, the induced voltage of each bunch is generated over a finite length, and it also introduces a phase shift in the induced voltage seen by the trailing bunches. The single-particle model can be extended by considering the induced voltage of a bunch with line density $\lambda(\hat{\tau})$ acting on the point-like bunches,

$$V_{\rm ind}(\tau_{bb} + \Delta \hat{\tau}) = -eN_b \int_{-\infty}^{+\infty} \lambda(\hat{\tau}') W'(\tau_{bb} - \hat{\tau}') d\hat{\tau}' \Delta \hat{\tau}.$$
 (5.11)



Figure 5.20 – Intensity threshold simulated for 72 point-like bunches with 630 MHz HOM impedance only in a single 200 MHz RF system with a voltage of 10 MV. The resonant frequency is shifted from 630 MHz by \pm 20 MHz. Colours represent the maximum amplitude of the bunch position oscillations, normalized by the average one.

In this case the instability growth time for the *i*-th bunch becomes

$$\frac{1}{\text{Im}(\omega)_{i}} = \left[\text{Im}\sqrt{\omega_{s0}^{2} - \mathscr{D}\sum_{k=0}^{i-1} \int \lambda(\hat{\tau}') W'[(i-k)\tau_{bb} - \hat{\tau}']d\hat{\tau}'} \right]^{-1}$$
(5.12)

Assuming a Gaussian bunch with a bunch length of 1.65 ns, the expression (5.12) can be calculated numerically for an intensity above the stability threshold, see Fig. 5.21. The picture is similar to the previous point-like bunch model. The asymmetry between odd and even 20 MHz lines is comparable, indicating that synchrotron intra-bunch motion plays a significant role in determining the stability threshold for a bunch train. Indeed, simulations at flat top with realistic bunches of 1.65 ns and the HOM impedance only, presented in Fig. 5.22, exhibit the same asymmetry as the simulations with the full longitudinal impedance model of the SPS shown in Fig. 5.17. The Figure 5.22 shows the threshold for the dipole oscillations, and a similar picture was obtained for the threshold of quadrupole oscillations. The stability improvement is the largest for resonant frequencies at 620 MHz and 640 MHz. At odd multiples of 20 MHz, the symmetry is similar to the point-like bunch model but at even values the shape of the stability threshold is reversed. These simulations indicate that the impedance of the 630 MHz HOM is the main source of instability and the interplay with other impedance sources does not play a significant role.

The simulations also show that the increase of intensity threshold, observed for a train of 72



Figure 5.21 – Instability growth time calculated for a train of point-like bunches and an induced voltage generated by a Gaussian bunch of length 1.65 ns. An intensity above the threshold is used $(4.3 \times 10^{11} \text{ ppb})$ and the sum in Eq. (5.12) is truncated for different lengths of interaction between bunches.



Figure 5.22 – Intensity threshold for 72 realistic bunches of 1.65 ns length simulated with the impedance of the 630 MHz HOM only. A single RF system at 200 MHz with a voltage of 10 MV is used. The resonant frequency is shifted from 630 MHz by \pm 20 MHz. Colours represent the maximum amplitude of the dipole oscillations of the last bunch in the train normalized by the average amplitude.

bunches, is principally due to the change in HOM frequency. Similarly to the case of a ring filled with equally spaced bunches, if the resonant frequency overlaps with a beam spectrum line or sits exactly between them, the bunches experience zero growth rate of instability.

A shift of the mode frequency in the positive direction, toward the spectrum line at 640 MHz, is favourable for beam stability but increases the heat load. On the contrary, a shift in the negative direction would have no detrimental effect on the heating but, one can see that the frequency band around 620 MHz where the stability is improved, is very narrow. Since most of the mode energy is stored in the cavity volume, a sufficient detuning is difficult to be achieved by means of RF couplers [57].

Note that, if the bunch spacing is increased to 50 ns—one of the potential solutions under consideration if the LHC suffers e-cloud effects—the 630 MHz band becomes a region of higher stability.

The point-like bunch model is able to account for the large gain in stability observed at resonant frequencies close to 620 MHz and 640 MHz for bunch trains but has difficulties to reproduce some of the finer details observed with real bunches. Nevertheless, for odd values of 20 MHz harmonics, the threshold is similar to the point-like bunch model.

The possible increase of the stability threshold above the LIU baseline by increasing the voltage

ratio between the two RF systems in double RF is explored in the next section.

5.5 Double RF System at Flat Top after LIU Upgrades

Already nowadays, the use of the fourth harmonic RF system of the SPS is a very efficient way to improve the beam stability. As explained in Section 3.4.2, the 800 MHz RF system increases the synchrotron frequency spread within the bunch, and enhances the Landau damping mechanism. However, in this case the synchrotron frequency distribution inside the bunch can also have a plateau where the first and even the second derivative of the synchrotron frequency goes to zero. In this case, as the analysis presented in Section 2.7 shows, Landau damping is lost and perturbations grow with time as $t^{1/2}$ or $t^{3/2}$, respectively.

The present situation in the SPS was analysed in Sections 4.2, 4.3, 4.4, and an increase in r was proposed. At a 200 MHz voltage of 10 MV, available after upgrades, the voltage ratio of 0.25 recommended at the flat top energy will not be achievable but the emittance increase for the same bunch length, due to the higher voltage, will improve the stability compared to the present case. The voltage delivered by the two TW cavities operating at 800 MHz can reach a maximum value of 1.7 MV (pulse mode) or 1.6 MV (no pulsing at f_0) at HL-LHC intensity (2.3 × 10¹¹ ppb) [79], which means that the maximum voltage ratio will be $V_{800}/V_{200} = 0.17$ or $V_{800}/V_{200} = 0.16$, respectively.



Figure 5.23 – Stability threshold of 72 bunches in a double RF system simulated at flat top with the longitudinal impedance model after LIU upgrades. The 200 MHz voltage is $V_{200} = 10$ MV. The nominal case $V_{800}/V_{200} = 0.1$ (blue) is compared to the cases with $V_{800}/V_{200} = 0.16$ (orange) and = 0.17 (green).

Figure 5.23 shows the stability threshold at flat top (450 GeV/c) for the two cases, compared to the LIU baseline case ($V_{800}/V_{200} = 0.1$). The second RF system is used in the bunch-shortening mode (phase from Eq. (3.12)). The improvement of the Landau damping by the fourth harmonic gives sufficient margins in terms of bunch length spread at flat top for beam stability at HL-LHC intensity.

Moreover, the 800 MHz RF system is equipped, today, with a feedback system which reduces the fundamental impedance of the 800 MHz cavities by -20 dB. The effect of this feedback system was not included in the longitudinal impedance model of the SPS since it was in the commissioning stage, but particle simulations for 72 bunches show a potential increase of the stability threshold when the highest voltage ratio of 0.16 is used, as shown in Fig. 5.24 and the stability is similar at HL-LHC intensity when the voltage ratio is 0.1.

The improvement of the beam stability should also be confirmed during the whole acceleration cycle for four nominal LHC batches, where the increase of the synchrotron frequency spread is less significant than at flat top and a large value of the ratio cannot be used during the first part of the acceleration.

The beam loading limitation should also be taken into account, leading to the fact that the full 200 MHz voltage of 10 MV will be available only for the LHC beam with a bunch intensity smaller or equal to 2.3×10^{11} ppb. Above this value, the available RF voltage is decreasing



Figure 5.24 – Stability thresholds in a double RF system at flat top with (dashed) and without (solid) the feedback system around the 800 MHz TW cavities (-20 dB). The longitudinal impedance model after LIU upgrades was used. The 200 MHz voltage is $V_{200} = 10$ MV. The nominal case where $V_{800}/V_{200} = 0.1$ (blue) is compared to the ultimate case where $V_{800}/V_{200} = 0.16$ (orange).

rapidly with the intensity. This aspect is treated in the next section.

5.6 Stability Threshold With Beam Loading Limitation

The effect of the beam loading on the RF voltage available to the beam can be included in simulations using Eq. (3.4). Part of the power is also used by the OTFB and the FF systems to compensate for the voltage induced by the beam at the fundamental mode (200 MHz). Figure 3.3 shows the available maximum voltage as a function of the bunch intensity. These values were used in simulations presented in this section to limit the voltage as a function of the bunch intensity assuming 4 LHC batches of 72 bunches spaced by 25 ns. The residual beam loading, which is not compensated by the OTFB and the FF systems, is taken into account by using, in simulations, the fundamental 200 MHz longitudinal impedance reduced by -26 dB. The corresponding stability thresholds are shown in Fig. 5.25 for the baseline case and the optimised case with a voltage ratio between the two RF systems of 0.16. The stability thresholds cross at a bunch intensity of 2.3×10^{11} ppb, since the 200 MHz voltage is 10 MV with and without beam loading limitation. For lower intensity, the stability threshold increases when



Figure 5.25 – Stability thresholds of 72 bunches simulated at flat top in a double RF system with the longitudinal impedance model after LIU upgrades. The baseline case with the 200 MHz voltage of 10 MV and a voltage ratio of 0.1 (blue solid line) is compared to the case where the beam loading limitation is included with the maximum voltage available at given intensity and a voltage ratio of 0.1 (blue dashed line), see also Fig. 3.3. The optimal case with a voltage ratio of 0.16 (orange solid line) is also included and compared to the case where the beam loading limitation is taken into account for the 200 MHz voltage and the voltage ratio is 0.16 (orange dashed line). A running average is applied to the stability thresholds in the case including beam loading limitation.

the voltage is a function of the bunch intensity. Indeed, the voltage is higher than 10 MV for $N_b < 2.3 \times 10^{11}$ ppb and the longitudinal bunch emittance is larger (for the same bunch length) compared to the case where $V_{200} = 10$ MV in all the intensity range. The stability threshold is reduced above HL-LHC intensity since the voltage available for the beam decreases quickly. The optimal case of a double RF system with a voltage ratio of 0.16, presented in Section 5.5, is also included in Fig. 5.25 and compared to the case where the beam loading limitation is included but the 800 MHz voltage is fixed to its maximum value available of 1.6 MV. This value is used since the voltage ratio can be increased, due to a smaller 200 MHz voltage above the HL-LHC intensity, which increases further the stability threshold. The effect of the beam loading is the flattening of the dependence on bunch length of the stability threshold with the fixed point where the bunch intensity is 2.3×10^{11} ppb when the voltage ratio is kept constant.

5.7 Conclusion

In this chapter the intensity limitations remaining after LIU upgrades were studied. The 200 MHz RF system of the SPS will be upgraded to reduce its impedance and to increase the available RF power and voltage. At HL-LHC intensity, a maximum RF voltage of 10 MV will be available at flat top. According to studies, for the nominal bunch length of 1.65 ns at extraction, the matched bunch emittance should be increased from the value at injection (0.35 eVs) to 0.57 eVs. The stability threshold of 72 bunches spaced by 25 ns will be raised by 50%, but this is nevertheless not sufficient to ensure beam stability of the HL-LHC beam with \sim 10% bunch length spread.

To improve further the LHC beam stability, the longitudinal impedance of the SPS ring should be reduced. The longitudinal impedance sources, giving the lowest stability thresholds, were identified in Chapters 3 and 4 to be the HOM at 630 MHz of the 200 MHz RF system and the impedance of the QF-type of vacuum flanges in the 1.4 GHz range. These flanges will be shielded during LS2 and their impedance in the 1.4 GHz range will be brought to a level invisible by the HL-LHC beam, now comparable to other SPS impedance sources as presented in Fig. 5.11.

Concerning the damping of the 630 MHz HOM, we identified that a reduction of its impedance by a factor of three is needed to assure beam stability at HL-LHC intensity, which was included in the baseline impedance reduction. Taking into account that the 630 MHz HOM is already heavily damped and further impedance reduction is difficult to achieve, other ways of reducing the effect of the 630 MHz HOM was investigated from a beam dynamic point of view in Section 5.4.3 using particle tracking simulations and a simplified analytical model. The effect on beam stability of a resonant frequency shift of the 630 MHz HOM was studied and we found that this HOM is presently in an asymmetric frequency region of reduced stability. It was found that for a bunch train, similarly to the case of a ring filled with equally spaced bunches, if the resonant frequency overlaps with a beam spectrum line (multiple of 40 MHz) or is exactly in between, the growth rate of the instability is zero.

Chapter 5. Longitudinal Beam Stability in the SPS after LIU Upgrades

A shift of the resonant frequency (630 MHz) towards these beam spectrum lines increases the stability. However, even if the positive effect of the HOM frequency shift is confirmed, in reality, the possibilities of shifting the resonant frequency are limited. A frequency shift in the 640 MHz direction is beneficial for beam stability as long as the HOM damper can handle the increased heat load. On the other side, the region of increased stability toward the 620 MHz notch is very narrow and a sufficient frequency shift cannot be achieved by means of RF couplers. Moreover, the natural spread of frequencies between the different cavities was measured to be around 100 kHz, which would not make any significant improvement of beam stability. Therefore other ways of increasing the stability threshold were examined.

It was shown in Section 5.4 that an impedance reduction of the MKP kickers or the sector valves increases the stability threshold, which could give sufficient margins (with $\sim 10\%$ bunch length spread) for beam stability of the HL-LHC beam. The technical solutions for both options already exist in case extra budget will be available.

Finally, it was shown that beam stability can also be greatly improved by optimization of the use of the SPS double RF system. For a 200 MHz voltage of 10 MV, the voltage ratio between the two RF systems can reach a maximum of 0.16. The limitations due to a plateau in the synchrotron frequency distribution for bunches with an average emittance of 0.6 eVs were analysed and it was shown that the synchrotron frequency spread can be increased without loss of Landau damping. The stability threshold increases even beyond the scope of the HL-LHC project, but at that point the beam loading limitations play an important role as shown in Section 5.6. Below a bunch intensity of 2.3×10^{11} ppb, the RF voltage can be increased above 10 MV, which increases the stability threshold for a given bunch length, but above the HL-LHC intensity the available RF voltage drops quickly below 10 MV. This effect flattens the intensity threshold of instability, as a function of bunch length, as shown in Fig. 5.25.
6 Lower-Harmonic RF System in the SPS

In the PS, the LHC proton beam is produced using many different RF systems (10 MHz, 20 MHz, 40 MHz, 80 MHz and 200 MHz). Before extraction, bunches are rotated in the longitudinal phase space in the 40 MHz RF system to reduce their length, before the transfer into the SPS 200 MHz RF system (see Section 3.5). Due to the relatively large nominal longitudinal emittance, a significant population of the bunch tails is injected close to the SPS RF separatrix. After filamentation, the particles fill the whole RF bucket, independent of the capture voltage, and particle losses in the SPS are observed during flat bottom and at the start of acceleration. The transmission from the PS to the SPS also degrades with intensity, and this poses a serious limitation when attempting to reach the high intensity goal of the HL-LHC project.

To reach a bunch length of 1.65 ns at SPS flat top, the 200 MHz SPS RF system is necessary. The bunch length before rotation in the PS is 14 ns [29] and supposing we would reach the bunch length of 1.65 ns in a 40 MHz RF system adiabatically, it would require a voltage technically not reachable, even at an energy of 450 GeV/c, since the bunch length scales like [86]

$$\tau \sim \left(\frac{\eta}{E_s V_{\rm RF}}\right)^{-1/4}.\tag{6.1}$$

Therefore, the transfer from the 40 MHz RF system of the PS to the 200 MHz RF system of the SPS is needed but it can be achieved, without bunch rotation, through an intermediate step with an additional lower-harmonic RF system in the SPS.

Even though a longitudinal coupled-bunch instability has been known to limit the beam intensity in the PS, it was demonstrated recently (in 2018) that the PS with the present transfer scheme (bunch rotation) can deliver 72 bunches with nominal emittance (0.35 eVs) and an intensity up to the HL-LHC one [87]. However, due to beam loading in the 200 MHz RF system of the SPS (even after LIU-SPS RF upgrade), particle losses cannot be avoided, if this transfer scheme remains [6]. From operational experience with present intensities, the average longitudinal emittance should not exceed 0.4 eVs at injection [7], but particle losses increase significantly for HL-LHC intensity [75].

To reduce losses for the HL-LHC beam, a lower-harmonic RF system could be installed in the SPS to capture the beam. Bunches without rotation in longitudinal phase space and with larger longitudinal emittance could then be injected while reducing the bunch population close to the RF separatrix. This would reduce in the SPS the losses related to the present bunch shape. This scenario of the PS-SPS beam transfer has been considered in the past, when the injector chain was in preparation for the LHC beam [88]. Beam loading in the 200 MHz RF system and coupled-bunch instabilities at flat bottom were considered as too serious limitations, and finally, the scenario with bunch rotation in the PS had been preferred. Presently, the SPS 200 MHz RF system is undergoing a significant upgrade which, together with other mitigation measures foreseen by the LIU project, allow the feasibility of this loss mitigation scheme to be revisited. Indeed, the upgraded (shortened) 200 MHz cavities will have less impedance. The Landau RF system at 800 MHz is also used to improve beam stability. In this Chapter, the scenario with a lower-frequency capture RF system as a loss mitigation scheme for the SPS, is discussed in detail.

First, the choice of the RF frequency of the new RF system is discussed. The bunch distributions at the PS flat top (26 GeV/c), before injection to the SPS, are treated in Section 6.2. In the third section, the beam stability at the SPS flat bottom energy (26 GeV/c), in the lower-harmonic RF system, is studied. Preliminary results for stability of 72 bunches at SPS flat top, simulated including a possible impedance of the new RF system, are also obtained. In Section 6.4, the injection of these bunches into the SPS is considered including intensity effects. An optimal capture voltage minimizing the uncontrolled emittance blow-up is found. The transfer to the main 200 MHz RF system, to allow the acceleration and the production of short enough bunches for the LHC, is discussed in the last section.

6.1 Choice of RF Frequency

The first constraint for the choice of frequency of the new RF system is the bunch spacing of 25 ns of the LHC beam which restricts the frequencies to multiples of 40 MHz. If a bunch spacing of 50 ns was to be used in the future, the frequency of the new system would still be valid. Secondly, the new SPS capture system must also provide sufficient bucket length without irredeemably degrading beam stability for bunches extracted without rotation in longitudinal phase space.

From the PS side, measurements at nominal LHC intensity have shown that a single bunch can be shrunk adiabatically to the length of 6 ns with the current RF hardware [87]. However, during this process, an uncontrolled emittance blow-up was observed, and the final bunch length of 6 ns still has to be demonstrated for the nominal HL-LHC batch of 72 bunches. For a given emittance, the bunch length at the SPS injection should be as small as possible to maximize the low-frequency RF voltage and to minimize the relative effects of the voltage induced at 200 MHz. In practice a bunch length of about 7 ns could still be acceptable. In the SPS, the only RF frequencies assuring a sufficient bucket length for the 6 ns PS bunches (or

larger) are 40 MHz and 80 MHz. The maximum bucket and bunch lengths for a bucket filling factor in momentum $q_p = 0.85$ are shown in Fig. 6.1 (left). The bunch length is computed for a full emittance. For higher frequencies (> 80 MHz), the bunch population close to the RF separatrix will be again significant and particles can be lost at the flat bottom energy or at the beginning of the acceleration.

For a fixed bunch emittance, beam stability is reduced in the 40 MHz and 80 MHz RF systems compared to the current 200 MHz RF system. Indeed, the threshold of coupled-bunch instability (Eq. (2.112)) and the loss of Landau damping threshold, $N_{\text{th}}^{\text{LLD}}$, defined in Ref. [19], both reduce with the harmonic number *h*, since the relative synchrotron frequency spread is proportional to h^2 . For given energy and machine optics (transition energy), the two stability thresholds scale as

$$N_{\rm th}^{\rm CBI} \propto \frac{\epsilon^2 h^2}{\tau},$$
 (6.2)

and

$$N_{\rm th}^{\rm LLD} \propto \epsilon^2 h^2 \tau.$$
 (6.3)

Reducing the harmonic number thus degrades the beam stability. For a constant bunch length, both thresholds scale with the parameter

$$S = \epsilon^2 h^2. \tag{6.4}$$

Its dependence on the RF frequency is shown in Fig. 6.1 (right) for two longitudinal emittances (nominal and 0.5 eVs). In particle simulations for 72 bunches, with the effect of the OTFB and the FF systems included, the threshold of the coupled-bunch instability at flat bottom, in the present double RF system with a voltage of 4.5 MV at 200 MHz and 0.45 MV at 800 MV, is above 4×10^{11} ppb. The scaling of the stability threshold predicts that it is reduced almost by a factor



Figure 6.1 – Left: bucket length and maximum full bunch length for a bucket filling factor in momentum $q_p = 0.85$ as a function of the RF frequency (multiples of 40 MHz). The horizontal dashed line indicates the minimum bunch length obtained in the PS for a single bunch without rotation. Right: scaling of stability thresholds at SPS flat bottom as a function of the RF frequency. The parameter $S = \epsilon^2 h^2$ is normalized to 0.35 eVs in the 200 MHz RF system.

10 in a 40 MHz RF system. Only an 80 MHz RF system with stabilization measures would allow for sufficient beam stability at SPS flat bottom for the HL-LHC beam.

Before investigating the beam stability and the injection into the SPS by means of particle tracking simulations, the new production scheme of PS bunches, by adiabatic procedure, is discussed in the next section.

6.2 Generation of Bunches in the SPS Injector

The longitudinal beam stability at SPS flat bottom strongly depends on the particle distribution of bunches extracted from the PS. This is valid for the current SPS operation, where the bunch rotation in the PS imposes a significant bunch population at large synchrotron amplitude. Accurate particle distribution is also important in the scenario without bunch rotation in the PS, and with a lower-harmonic RF system, for bunch capture, in the SPS. Indeed, particles in the bunch could be lost from the main 200 MHz RF bucket during transfer from one RF system to the other. Therefore, the PS bunch generation is a critical part to be included simulations, and a good representation of the particle distribution in operation is necessary to depict the variety of bunches that can be extracted. The bunch tails in the PS are difficult to measure and the exact particle distribution in operation after LIU upgrades cannot be known *a priori*. Therefore, different particle distributions are considered, with emittances in the range of interest and the tails going from practically no tails (waterbag model) to those close to a Gaussian distribution.

At PS flat top (26 GeV/c), two RF systems are used, one operating at 40 MHz and the other at 80 MHz. In operation, before bunch shortening, the 40 MHz voltage is 50 kV whereas the 80 MHz voltage is zero. In this configuration the bucket area is 0.6 eVs, like at SPS flat bottom in nominal operation. The voltage of both RF systems can be increased adiabatically to a maximum of 600 kV to reduce the bunch length. Compared to the bunch rotation, this process removes the S-shape of the particle distribution and decreases the bunch population at large synchrotron amplitude. However, it cannot produce short enough bunches (6 ns measured with a single bunch [87]) for the 200 MHz RF bucket of the SPS and a lower-harmonic RF system is needed to capture these bunches. The voltage program used in operation for adiabatic bunch length reduction at flat top is presented in Fig. 6.2, where the 0 cycle time corresponds to arrival at flat top.

In simulations, at the end of the adiabatic bunch-length reduction process, bunches are considered as matched to the RF bucket. The particle distribution at PS flat top (26 GeV/c) is, therefore, matched to the RF bucket of the double (40 MHz and 80 MHz) RF system with a voltage of 600 kV in both RF systems, without intensity effects. The particle distribution is generated using the binomial function from Eq. (2.108) for a single bunch with a large number of macroparticles (3.6×10^7) in phase space, and subsets of 10^6 macroparticles are randomly chosen to create a multi-bunch beam in the SPS. The parameter μ of the binomial distribution controls the extension of the bunch tails. When it is large ($\mu > 5$), the particle distribution



Figure 6.2 – RF voltage programs at 40 MHz (blue) and 80 MHz (orange) for adiabatic bunch shortening at the PS flat top (26 GeV/c).

tends to a Gaussian distribution. The parameter J_0 (action) of the binomial distribution is related to the full longitudinal bunch emittance (rescaled by 2π). This emittance corresponds to the area enclosed by the outermost trajectory of particles in phase space.

Usually, in operation and simulations, the emittance is computed using Eq. (2.70) and the unperturbed Hamiltonian (no intensity effects). The FWHM bunch length rescaled to 4σ assuming a Gaussian distribution (Eq. (2.71)), $\tau_{4\sigma}$, is used to define the integration limits in Eq. (2.70)

$$\phi_1 = \phi_s - \frac{\omega_{\text{RF}}\tau_{4\sigma}}{2}, \qquad \phi_2 = \phi_s + \frac{\omega_{\text{RF}}\tau_{4\sigma}}{2}. \tag{6.5}$$

However, this definition is not ideal for comparison of bunches with different μ . In the case of a Gaussian bunch, the FWHM bunch length, rescaled to 4σ , contains 95% of the particles. This is, however, not true for $\mu < 5$. We define the bunch length τ_{95} corresponding to the emittance ϵ_{95} which contains 95% of the particles, independent of the value of μ . This approach is also used in the PS measurements [89]. These quantities are obtained in simulations by computing the unperturbed Hamiltonian of every particle and extracting the emittance value which contains the correct number of particles. This method will be used throughout the rest of the chapter to define the bunch length and emittance.

Figure 6.3 shows examples of bunch profiles with different μ from Eq. (2.108), for a longitudinal bunch emittance $\epsilon_{95} = 0.5$ eVs, at PS flat top. In previous SPS operation, bunches with $\mu = 0.5$ are not observed, and they will not be considered below. The value $\mu = 1.5$ is close to what is observed at SPS flat top, but the extension of the bunch tails, at flat bottom, in this scenario of



Figure 6.3 – Line density of matched bunches at PS flat top (26 GeV/c), normalized such that $\int \lambda(\hat{\tau}) d\hat{\tau} = 1$, with $V_{40} = 600$ kV and $V_{80} = 600$ kV. The binomial function, with bunch emittance $\epsilon_{95} = 0.5$ eVs, is used for different parameters μ between 0.5 and 5.0.

an 80 MHz capture system in SPS, are not known *a priori*. Values of μ between 1.5 and 3.5 were used in the analysis. The particle distributions in phase space are shown in Fig. 6.4 for $\mu = 1.5$ (a) and $\mu = 3.5$ (b). The white line is the trajectory in phase space which encloses 95% of the particles and they both corresponds to $\epsilon_{95} = 0.50$ eVs. For particle distribution with larger μ , particles outside the 95% region are spread over larger amplitude in phase space, which makes the transfer to the 200 MHz bucket more difficult. First, the beam stability in the SPS is studied below.

6.3 Beam Stability in the Lower-Harmonic RF System

In the scenario considered in this chapter, the SPS RF system consists of RF cavities operating at 80 MHz and 200 MHz. The limitation of the maximum voltages available due to the beam loading is not taken into account. The main 200 MHz RF system of the SPS is needed to transfer short enough bunches to the 400 MHz RF bucket of the LHC.

The scenario of an 80 MHz RF system for bunch capture in the SPS was rejected in the past [88] because of beam instabilities, mainly due to the 200 MHz impedance, which were considered to be difficult to mitigate. Since then, the impedance of the machine was significantly reduced and the LIU-SPS RF upgrade will improve the beam control further. However, still as suggested by Fig. 6.1 (b), the beam stability threshold in the single 80 MHz RF system is expected to be significantly lower than in the 200 MHz RF system.



Figure 6.4 – Particle distribution in the longitudinal phase space for bunches generated at PS flat top (26 GeV/c) in $V_{40} = 600$ kV and $V_{80} = 600$ kV with a longitudinal emittance $\epsilon_{95} = 0.5$ eVs. The parameters of the binomial distribution are $\mu = 1.5$ (a) and $\mu = 3.5$ (b). The red line corresponds to the RF separatrix. The white line is the trajectory which encloses 95% of the particles. The colours of the particle distributions represent the density of particles (in arbitrary units), with red being the densest and blue the lightest.

The beam stability in a single 80 MHz RF system was studied in particle simulations with 48 bunches. This batch length provides results close to those from the simulations of 72 bunches but with a reduced simulation time. The particle distributions generated in the PS without bunch rotation and described in Section 6.2 were used. For beam stability analysis, the capture voltage at 80 MHz was fixed to 0.7 MV (see Section 6.4). Larger amplitudes of the 80 MHz voltage at injection would introduce an uncontrolled emittance blow up after filamentation, which is not desirable for transfer to the 200 MHz RF system later. However, this value of the 80 MHz voltage can be adjusted if necessary, see Section 6.4. Bunches with $\mu = 0.5$ are excluded from consideration because they are very unstable and their distribution is not realistic for operation.

Figure 6.5 shows the stability threshold for the 48 bunches in the single 80 MHz RF system after the 10 s of flat bottom (26 GeV/c). The threshold is fit with a square function of the bunch emittance ϵ_{95} , according to Eq. (6.4). Already the nominal intensity of the LHC beam $(1.15 \times 10^{11} \text{ ppb})$ is close to the instability threshold and, if the impedance of the 80 MHz RF system is taken into account, the threshold could further decrease (see below). Even for large emittances (0.5 eVs), the intensity limit is not sufficient for the HL-LHC beam.

Nevertheless, we studied the possibility that the 200 MHz RF system can be used as a Landau system for the 80 MHz RF system to stabilize the beam. As explained in Section 4.2, the spread of the synchrotron frequency is important to improve the beam stability, but also the shape of the synchrotron frequency distribution within the bunches. For the SPS operation in the double 200 MHz and 800 MHz RF system, it was shown that a voltage ratio between the two



Figure 6.5 – Stability threshold of 48 bunches in a single 80 MHz RF on flat bottom (26 GeV/c) of 10 s as a function of the emittance at injection containing 95% of the particles, ϵ_{95} . The 80 MHz voltage is 0.7 MV in all cases. The stability threshold for the particle distribution from Eq. (2.108) with $\mu = 1.5$ (green) is compared to the cases where $\mu = 2.5$ (orange) and $\mu = 3.5$ (red).

RF systems of 0.1 should be maintained at flat bottom. However, in that case the ratio of the frequencies of the two RF systems was 4 whereas in the scenario with 80 MHz and 200 MHz this ratio is 2.5 as follows from Eq. (6.6). As explained in Section 2.3, the maximum 200 MHz voltage, for use as a Landau system, is

$$V_{200} = \frac{h_{80}}{h_{200}} V_{80}.$$
(6.6)

The maximum ratio h_{80}/h_{200} is 0.4 in this case and higher values would create higher frequency buckets inside the 80 MHz bucket, like ratios higher than 0.25 for the double 200 MHz and 800 MHz RF system. The synchrotron frequency distribution for the double 80 MHz and 200 MHz RF system is shown in Fig. 6.6 for the maximum ratio defined by Eq. (6.6) in the bunch-shortening and bunch-lengthening modes. It is also compared to the single 80 MHz RF system case. In bunch-lengthening mode, the synchrotron frequency distribution becomes flat within bunches with a small emittance. The bunch-shortening mode is proposed to increase the frequency spread for bunches up to an emittance of 0.50 eVs.

Simulations of the stability threshold in the double 80 MHz and 200 MHz RF system were done for the same beams as in the single RF case. The 80 MHz voltage was 0.7 MV and the 200 MHz voltage of 0.28 MV is defined by Eq. (6.6). This low 200 MHz voltage is difficult to



Figure 6.6 – Synchrotron frequency distribution in the single 80 MHz RF system (blue) and the double (80 MHz and 200 MHz) RF system in bunch-shortening mode (orange solid) and bunch-lengthening mode (orange dashed). The two vertical lines indicate emittances of 0.35 eVs and 0.50 eVs.

control in the present SPS operation but after the RF upgrade the hardware will have no lower limitation in terms of total voltage [90]. The 800 MHz RF system with $V_{800} = 0.1 \times V_{200}$ in the bunch-shortening mode with respect to the 200 MHz RF system was also tested in simulations, since it is essential in preserving beam stability throughout the cycle with 200 MHz as a main accelerating system, see the previous chapters. However, simulations without the 800 MHz show no impact of this system on the beam stability or the bunch transfer into the 200 MHz RF bucket at flat bottom due to the very low voltage of 28 kV. Such a voltage is very difficult to control in the SPS operation in the presence of the beam loading and the case of a triple RF system is not presented in what follows. The stability thresholds are shown in Fig. 6.7.

A significant improvement is indeed observed in the double RF system, allowing the HL-LHC intensity to be reached for bunches with $\mu = 1.5$ and emittances ϵ_{95} larger or equal to 0.35 eVs. To determine the optimal beam parameters at HL-LHC intensity, a study of the effect of the 80 MHz impedance and other RF settings (voltage ratio and phase) would be necessary, but beam stability is demonstrated with sufficient margins for the different beams that will be accelerated after LS2 (longitudinal bunch emittances between 0.35 eVs and 0.5 eVs), since the parameter μ is close to 1.5 in current operations.

A second important point concerning the beam performance with the new 80 MHz RF system is the impact of its impedance on the beam stability during the full acceleration cycle. To conclude this section, we analyse this effect at SPS flat top, where the stability threshold is



Figure 6.7 – Stability threshold of 48 bunches in a double 80 MHz and 200 MHz RF system on flat bottom (26 GeV/c) of 10 s as a function of the emittance at injection containing 95% of the particles, ϵ_{95} . The 80 MHz voltage is 0.7 MV in all cases and the 200 MHz voltage is defined by Eq. (6.6). The stability threshold for the particle distribution from Eq. (2.108) with $\mu = 1.5$ (green) is compared to the cases where $\mu = 2.5$ (orange) and $\mu = 3.5$ (red).

the lowest, using as a model the existing 80 MHz RF cavity in the PS, to estimate the possible contribution of the impedance of such a system in the SPS. The 80 MHz voltage should reach 3 MV as it will be explained in Section 6.5, which requires ten PS 80 MHz cavities (with spare cavities). The longitudinal impedance of such a system is shown in Fig. 6.8 (left) [91].

Particle tracking simulations were done for 72 bunches at flat top in the double 200 MHz and 800 MHz RF system, using the longitudinal impedance of the SPS after LIU upgrades where the impedance from Fig. 6.8 (left) was added. Simulations showed that the stability threshold at flat top depends weakly on the HOMs of the 80 MHz cavities. Nevertheless, the fundamental impedance can have an important impact and a dedicated feedback system to reduce this impedance is necessary. The current hardware in the PS can reduce the 80 MHz impedance by -40 dB, but this system cannot necessarily be installed in the SPS tunnel due to space limitations. The stability threshold at a bunch length of 1.65 ns was studied in simulations as a function of the feedback reduction and results are shown in Fig. 6.8 (right). This result indicates that a reduction by the feedback of ~-40 dB would indeed also be necessary in the SPS. Even if this part of the study is very preliminary, the need of a large impedance reduction at 80 MHz should be taken into account in the design of the new RF system.

In the next section, the injection into the SPS including intensity effects is discussed. The value of 80 MHz voltage of 0.7 MV used in this section will also be justified.



Figure 6.8 – Left: Total impedance of the 80 MHz cavities from the PS including HOMs [91], providing a maximum of 3 MV in the SPS (10 cavities). The effect of the fast cavity feedback has been measured with the beam and the effective reduction is -40 dB (included in the plot). Right: Stability threshold for 72 bunches with length of 1.65 ns at SPS flat top in the double 200 MHz and 800 MHz RF system as a function of the feedback reduction at 80 MHz with the impedance from the left figure added to the longitudinal impedance of the SPS after LIU upgrades. The horizontal dashed line indicates the value of the threshold without 80 MHz impedance.

6.4 Simulations of Injection Into the SPS With Intensity Effects

The matched 80 MHz voltages V_{80} for the different emittances and bunch lengths are in the interval of (0.6–0.7) MV, but adjustments were required due to the uncontrolled longitudinal emittance blow-up along the batch. Indeed, there is no direct matching from the PS 40 MHz and 80 MHz RF systems to the SPS RF systems. To avoid substantial losses in the neighbouring 200 MHz buckets during re-bucketing, the bunch emittance should be kept as small as possible. At HL-LHC intensity, the induced voltage in the SPS can reach a peak value of 0.2 MV for the distributions considered, which can increase the bunch emittance in an uncontrolled way along the batch.

The capture voltage at 80 MHz, which minimizes the emittance blow up after capture, was determined using simulations. Batches of 30 bunches spaced by 25 ns are injected in the SPS with distributions shown in Section 6.2. The length of the batch is chosen to take into account the effect of the induced voltage from the beam loading, which saturates after 24 bunches. The full longitudinal bunch emittances are in the range from 0.3 eVs to 0.5 eVs. The parameter μ is in the range from 0.5 to 3.5. The bunches are centred individually in their SPS RF bucket with intensity effects included. In operation this matching is controlled by the low-level RF phase loop. In present operation, the matching cannot be perfect, since it is based on average bunch positions in the batch, situation that will be improved after the SPS RF upgrade. In simulations, it was assumed to be perfect. The feasibility of the scenario was studied with this perfect matching, remembering that a mismatch in operation can increase losses and the longitudinal emittance blow-up along the batch.

In simulations, the longitudinal impedance model after LIU upgrades described in Section 5.3 was used. The 80 MHz capture voltage is varied between 0.4 MV and 1.5 MV. The 200 MHz voltage was fixed by Eq. (6.6). Bunches are injected at flat bottom energy (26 GeV/c) and the simulations last enough synchrotron periods to let the process of filamentation finish (without intensity effects it is about 250 periods). The average bunch length and the bunch emittance computed for all bunches in the train are shown in Fig. 6.9 for a full emittance of 0.50 eVs in the PS. The two cases shown are for $\mu = 1.5$ and $\mu = 3.5$. For the different bunches simulated, the optimal 80 MHz voltage for bunch capture is in the range (0.6–0.8) MV. It is observed that for low values of the voltage ($V_{80} < 0.5$ MV), bunches are unstable and fill a large part of the RF bucket. In simulations for higher voltages ($V_{80} > 0.8$ MV), bunches are stable with the 200 MHz RF system included, but their longitudinal emittance is blown up due to the mismatch. This



Figure 6.9 – The 95% bunch emittance (right) and the corresponding bunch length (left) as a function of the capture voltage V_{80} for a batch of 30 bunches in the SPS after filamentation. The full emittance in the PS is $\epsilon = 0.5$ eVs. The parameter μ is 1.5 (a,b) and 3.5 (c,d). The emittance is interpolated around its minimum with a square function of the capture voltage at 80 MHz. The 200 MHz system is included in simulations with a voltage $V_{200} = 0.4 \times V_{80}$.

means that the 80 MHz capture voltage in the SPS in a range between 0.6 MV and 0.8 MV does not lead to significant emittance increase along the train for all distributions considered. An average value of 0.7 MV was used to study the stability at injection in the previous section and is also used in what follows.

In the next section, we will discuss the question of the transfer to the 200 MHz RF system of bunches injected in the 80 MHz RF system with the 200 MHz. The addition of the third RF system at 800 MHz with a small voltage does not make any significant difference for particle losses from the main 200 MHz bucket, compared to the double RF case (80 MHz and 200 MHz).

6.5 Transfer to the Main RF System

To obtain an average bunch length of 1.65 ns at SPS flat top, for injection into the LHC, the 80 MHz voltage alone should be 17.5 MV (if the bunch length is adiabatically reduced). Practically, this is not feasible in terms of budget, impedance and space along the SPS ring. Moreover, the beam stability is already at the limit at flat bottom and the stability threshold scales with the inverse of the synchronous energy, see Eq. (2.112). At flat top, the stability in the double 80 MHz and 200 MHz RF system would certainly not be sufficient even for the nominal LHC beam. The beam must be transferred to the 200 MHz RF buckets well before flat top. We have chosen to do the transfer to the 200 MHz RF system at flat bottom, where the voltage available for the beam is the highest, since the beam loading is the smallest thanks to long bunches.

The bucket length in the 80 MHz RF system of 12.5 ns is large with respect to the length of injected bunches of 6-7 ns. Particles cannot be lost from the bucket. However, during the rebucketing particles can be lost from the main 200 MHz RF bucket and can end in neighbouring buckets forming satellite bunches, harmful for LHC physics. If there are too many satellite bunches or, if the intensity stored in them is too high before arrival of the first bunch of the beam, they create losses in the LHC as they see a non-nominal field of the injection kicker. In between the main buckets, spaced by 25 ns, they create off-trigger events that pollute the data analysis of the experiments. From operational experience, the intensity stored in each satellite should not exceed the injected bunch intensity by a ratio of 10^{-3} [92].

To simulate the capture into the main 200 MHz bucket, 30 bunches spaced by 25 ns, with the particle distribution defined in Section 6.2, are injected at 26 GeV/c. The position of each bunch was matched to the centre of their RF bucket including intensity effects. The double RF system with voltages at 80 MHz and 200 MHz was used. The capture voltage at 80 MHz minimizing the uncontrolled emittance blow-up after capture was determined in Section 6.4. The optimum voltage is slightly different for distinct particle distributions with values between 0.6 MV and 0.8 MV. The 200 MHz voltage at injection is defined by Eq. (6.6).

The length of non-rotated PS bunches is too large to fit into the 200 MHz RF bucket of 5 ns. The RF voltage program for beam transfer at flat bottom reduces adiabatically the length of

the bunches. After capture and filamentation, the 80 MHz voltage (V_{80}) was increased over many synchrotron periods (> 50), to a maximum value V_{80}^{max} . This maximum 80 MHz voltage was varied to determine the value giving acceptable particle population in neighbouring buckets. During this first part of the process, we kept $V_{200} = 0.4 \times V_{80}$. Then, the voltage at 200 MHz was increased to 8 MV to complete the transfer, while V_{80} was reduced to zero. The full RF voltage program for transfer to the main 200 MHz bucket is shown in Fig. 6.10 (a). The 200 MHz voltage at the end of the process (8 MV) can be adjusted, but the particle population in satellite bunches is only weakly dependent on this value. Simulations were done using the longitudinal impedance model after LIU upgrades, presented in Section 5.3, for injected bunch intensity $N_b = 2.4 \times 10^{11}$ ppb. Figure 6.10 (b) shows an example of the average intensity (over 30 bunches) inside the 80 MHz and the 200 MHz RF buckets for an injected bunch emittance (containing 95% of particles), $\epsilon_{95} = 0.5$ eVs. The intensity was computed as the number of particles inside the 12.5 ns of the 80 MHz RF bucket and inside the 5 ns of the 200 MHz RF bucket, respectively. The bunch length τ_{95} during cycle is also presented. The number of particles lost from the main 200 MHz bucket is defined in the first part of the transfer when the 80 MHz voltage reaches its maximum value. The losses in the second part of the process are due to particles close to the 200 MHz separatrix that are not captured in the main 200 MHz bucket when V_{200} increases.

We define the relative particle losses, for each bunch (N_b^{lost}) as the difference between the intensity inside the main 200 MHz bucket at the end of the transfer, N_1 , and the bunch intensity



Figure 6.10 – (a) An example of voltage programs for bunch transfer from the 80 MHz to the 200 MHz RF bucket where $V_{80}^{\text{max}} = 2.5$ MV. (b) The average intensity in the 80 MHz RF bucket (black), compared to the average intensity in the main 200 MHz RF bucket (blue) for a batch of 30 bunches with $\epsilon_{95} = 0.5$ eVs ($\mu = 1.5$) and an injected bunch intensity $N_b = 2.4 \times 10^{11}$ ppb. The average bunch length along the batch is shown in green.

at injection N_0 , normalized by N_0

$$N_b^{\text{lost}} = \frac{N_0 - N_1}{N_0}.$$
 (6.7)

In simulations, the maximum 80 MHz voltage during the transfer, V_{80}^{max} , was varied between 1.5 MV and 3.5 MV to estimate the fraction of the bunch lost as a function of V_{80}^{max} . The cases for emittances of $\epsilon_{95} = 0.35$ eVs (a,c) and $\epsilon_{95} = 0.50$ eVs (b,d) and particle distributions with $\mu = 1.5$ (a,b) and $\mu = 3.5$ (c,d) are presented in Fig. 6.11. The particle population in satellite bunches is smaller for bunches with smaller tails ($\mu = 1.5$). For bunches with large tails ($\mu = 3.5$), if the longitudinal emittance ϵ_{95} exceeds some threshold value, the particle population in satellite bunches becomes larger than the acceptable value of $N_h^{\text{lost}} = 10^{-3}$ for



Figure 6.11 – Relative losses N_b^{lost} as a function of the maximum 80 MHz voltage during the beam transfer V_{80}^{max} for particle distributions with $\mu = 1.5$ (green) and $\mu = 3.5$ (red) and two bunch emittances (containing 95% of particles), $\epsilon_{95} = 0.35$ eVs (a,c) and $\epsilon_{95} = 0.50$ eVs (b,d). The horizontal dashed line on the four figures indicates the maximum relative losses acceptable for LHC. The losses are linearly interpolated between the simulated points.

LHC (and HL-LHC), independent of the maximum 80 MHz voltage.

Due to beam loading, losses increase along the batch. The relative losses N_b^{lost} of the last bunch in the train (30th) are the biggest and are used below as an indicator of the maximum particle population in satellite bunches after transfer to the main 200 MHz bucket. The relative losses N_b^{lost} as a function of the longitudinal bunch emittance ϵ_{95} after injection to 80 MHz are shown in Fig. 6.12 for different values of μ between 1.5 (flat top value in present operation) and 3.5 (maybe too pessimistic). The three cases, $V_{80}^{\text{max}} = 2.0$ MV, 2.5 MV, and $V_{80}^{\text{max}} = 3.0$ MV were analysed. The relative losses strongly depend on the injected longitudinal bunch emittance and the bunch tails. A maximum 80 MHz voltage of, at least, 2.5 MV would be necessary to recapture bunches with longitudinal bunch emittance $\epsilon_{95} = 0.50$ eVs and limited tails ($\mu = 1.5$),



Figure 6.12 – Relative losses N_b^{lost} as a function of the longitudinal bunch emittance ϵ_{95} after transfer into the 200 MHz RF bucket for three maximum 80 MHz voltages, $V_{80}^{\text{max}} = 2.0$ MV (a), 2.5 MV (b), and 3.0 MV (c) for particle distributions with $\mu = 1.5$ (green), 2.5 (orange), and 3.5 (red). The bunch intensity is $N_b = 2.4 \times 10^{11}$ ppb and the 80 MHz voltage at injection is 0.7 MV in all cases. Losses are linearly interpolated between the simulated points. The horizontal dashed line (black) indicates the maximum N_b^{lost} acceptable for LHC.

and keep the particle population in neighbouring buckets below the level of 10^{-3} . For larger values of maximum 80 MHz voltage, as shown in Fig. 6.12 (c), the gain in terms of relative losses is very limited. More losses can be expected for 4 batches of 72 bunches.

6.6 Conclusion

This analysis of the scenario of an 80 MHz RF system in the SPS for capture of the HL-LHC bunches has shown the feasibility of this new PS-SPS transfer scheme. The required beam stabilization in the low-harmonic RF system, considered in the past as a main limiting factor, can be achieved using the existing 200 MHz RF system in bunch-shortening mode.

Bunch capture and transfer to the 200 MHz RF system were studied in detail using macroparticle simulations of 30 bunches with different particle distributions including intensity effect. The 80 MHz voltage at injection must minimize the uncontrolled emittance blow-up without being too small due to beam loading, but the maximum voltage needed is defined by the transfer to the 200 MHz RF bucket and depends mainly on the bunch distribution. The drawback of this scenario is the high 80 MHz voltage required (2.5 MV), since large tails of the injected bunches with an emittance larger than 0.35 eVs could bring the number of particles in satellites to a level unacceptable for the LHC. Moreover, particle losses could increase, compared to the simulations presented here, for 4 batches of 72 bunches. It is, nevertheless, possible to remove a part of the bunch tails in the PS before injection into the SPS (bunch longitudinal shaving).

In comparison with the present transfer scheme (bunch rotation), the total number of particle losses before acceleration can potentially be reduced. In this scenario of an 80 MHz RF system for bunch capture in the SPS, simulations of acceleration including the impedance of the new RF system would be needed to confirm the gain. The impedance of the 80 MHz cavities could decrease the beam quality and it would have an impact during the cycle, especially at flat top, where the stability threshold is the lowest. Simulation results using the impedance of the PS 80 MHz cavities show that a direct feedback system would be necessary to achieve a reduction of the fundamental 80 MHz impedance by more than 35 dB. An additional RF system would also increase the complexity of the SPS operation due to additional RF manipulations.

The potential gain from such a system is a reduction of losses in the SPS assuming that the PS can deliver beams with sufficient quality but the longitudinal impedance of the system could be harmful for the LHC beam and further studies are necessary to confirm the advantages for the SPS.

Conclusion

In the CERN SPS, the present nominal LHC bunch intensity of 1.15×10^{11} ppb is already close to the limit that the machine can deliver. This thesis studied the longitudinal intensity limitations of the LHC proton beam in the SPS consisting of four batches of 72 bunches and possible mitigation measures to reach the HL-LHC bunch intensity which is twice higher than the nominal one. Detailed conclusions were made at the end of each Chapter and a brief summary is given below.

In Chapter 1, the introduction was given. The BLonD code was briefly presented. In the course of this thesis, it has been adapted for multi-bunch simulations and the runtime was decreased by using high-performance-computing resources and algorithm optimizations. This permitted to perform particle tracking simulations of a SPS batch of 72 bunches at flat top energy and 12 bunches during the complete acceleration ramp with a realistic bunch distribution in a reasonable time, which was not feasible in the past. It was an important step forward in the study of the LHC beam stability since simple analytic models do not fit the observations. Moreover, the simulations allow to investigate the effect of the unmatched bunch distributions (rotated bunches), to trace down the source of intensity limitations and instabilities, and to predict the future performance of the machine.

In Chapter 2, the necessary theoretical background was introduced. The main intensity limitations of the SPS were analysed in Chapter 3. They are the beam loading in the 200 MHz RF system, the longitudinal multi-bunch instabilities triggered during the acceleration ramp and the particle losses, increasing with the beam intensity. The present detailed longitudinal impedance model of the SPS was presented in Section 3.2. In Section 3.3, we established through particle simulations that this model is able to reproduce the bunch lengthening with intensity and the stability thresholds which are observed for a single bunch.

The beam measurements carried out with thorough data analysis demonstrated that the impedance of the 630 MHz HOM in the 200 MHz RF system is potentially responsible for the longitudinal coupled-bunch instability observed at the SPS flat top energy. In addition, it has been shown that there is an interplay between different machine impedances in the formation of the stability threshold, making the simulations essential in understanding the instability mechanism and planning of necessary impedance reduction. Indeed, we showed that the impedances of the QF flanges together with the HOM at 630 MHz have a significant impact

on the stability threshold for 72 bunches at the flat top energy, even though the theoretical threshold for coupled-bunch instability of the QF flanges is much higher than the one of the 630 MHz HOM.

In Chapter 4, we presented a rigorous comparison of particle simulations performed with the code BLonD and results of the data analysis of beam measurements carried out over many dedicated sessions using beams of 12 bunches. The results have confirmed the influence of the 630 MHz HOM on the stability threshold in the last part of the acceleration cycle and suggested that the intensity limitation at flat bottom and in the first part of the acceleration is determined by the fundamental impedance of the 200 MHz RF system. Our comparison between beam measurements and simulations also permitted to disentangle the effect of specific systems in the 200 MHz low-level RF control on the stability threshold during the ramp and to benchmark the longitudinal SPS impedance model. In particular, the effect of the one-turn-delay feedback and the feedforward systems on the stability threshold were studied using BLonD simulations since it would be very complicated to include them in analytical estimations. Moreover, the particle simulations with the longitudinal impedance model of the SPS reproduce well the stability threshold measured at flat top which gives confidence in the predictions for the future situation with 72 bunches.

In Section 4.2, the double RF operation used for beam stabilization was studied in particle simulations and compared with beam measurements. We have confirmed that not only the spread of the synchrotron frequency is important for the stability threshold but also the shape of the synchrotron frequency distribution within the bunches. For bunches with a longitudinal emittance containing a plateau in the synchrotron frequency distribution, the stability threshold is significantly reduced. Based on this fact, we obtained an optimized program during the cycle for the voltage ratio between the 200 MHz and 800 MHz RF systems to improve the beam stability during the whole cycle. This program was successfully tested in the SPS and stable 12 bunches of high intensity were injected and used in the LHC. We showed that the bunch-shortening mode is the best mode of operation in a double RF system in the SPS for large bunches (0.35 eVs and more). In this chapter, it was also established that the particle distribution in the longitudinal phase space, which arises from the bunch rotation in the PS, is reducing the beam stability and is crucial to reproduce the stability threshold correctly at the SPS flat bottom energy.

The successful benchmarking between particle simulations and beam measurements reinforced the possibility to use our simulation model to investigate the possible performance of the SPS after LIU upgrades in terms of beam stability. In Chapter 5, we established that a reduction of the HOM impedance at 630 MHz by a factor of 3 would be needed for beam stability at HL-LHC intensity. Since this impedance is already heavily damped by a series of RF couplers, a possible attenuation of the effect of this impedance by shifting the resonant frequency of the mode towards beam spectrum lines was also investigated. The obtained results are promising to improve the beam stability but difficult to apply in practice due to various limitations. We also evaluated different scenarios of further impedance reduction, which could give sufficient margins for the stability of the HL-LHC beam, taking into account the realistic spread of bunch length (around 10%) in present operation. The double RF operation, after RF upgrades, was also demonstrated to continue to be, together with the controlled emittance blow-up from the injected value to 0.57 eVs, a main mitigation measure to ensure the stability of the HL-LHC beam.

Finally, in Chapter 6, the scenario of a lower-harmonic RF system in the SPS to reduce particle losses by capturing the bunches from the PS obtained by an adiabatic process (without rotation), was studied. It was determined that only an 80 MHz RF system is suitable for this scenario. It was shown that the beam stabilization, which was considered as a main limiting factor for such a system in the past, can be achieved using the existing 200 MHz RF system in bunch-shortening mode. The voltages required for beam capture and transfer to the 200 MHz RF system were defined using macroparticle simulations. The drawback is the large 80 MHz voltage needed for the transfer of bunches from the 80 MHz to the 200 MHz RF bucket, which depends mainly on the details of the bunch distribution and the analysis can be refined when the new HL-LHC beam will be circulating in the machines. The effect of the 80 MHz impedance on the stability threshold at SPS flat top energy was also studied using the model of the existing 80 MHz PS cavities. The results have shown that the impact of the impedance of the HOMs is not significative but a feedback system is needed to reduce the fundamental 80 MHz impedance by ~-40 dB.

The main achievement of this thesis is that we showed by analytical methods and particle simulations including the complete SPS longitudinal impedance model and effect of the SPS low-level RF system, that the SPS intensity limitations to reach the HL-LHC intensity can be overcome by planned LIU upgrades. This was made possible by optimizations of the simulation code and the use of high-performance-computing resources. Thorough benchmarking of the simulations including the longitudinal SPS impedance model with beam measurements revealed a good agreement and tend to proof that the simulations are a reasonable representation of reality. Simulations with 72 bunches were necessary to assess the stability thresholds due to the large quality factor of certain impedances of the machine giving significantly different intensity limits with a smaller number of bunches. It was a major breakthrough in the study of the stability of the LHC beam in the SPS since it allowed to investigate the effect of each contribution to the SPS impedance independently and to evaluate the effect of the double RF system. The requirements for the impedance reduction campaign were obtained and an additional improvement of the beam stability due to optimized use of a double RF system was proposed.

A Resonator Models of the Longitudinal SPS Impedance Before LIU Upgrades

In this appendix, the parameters of the resonators for the longitudinal SPS impedance model before LIU upgrades are given. Resonators with a value of $R_{\rm sh}/Q$ smaller than 10 Ω and a resonant frequency larger than 2.5 GHz are not shown. The fundamental impedance of the TW structures, which are not described by resonators, was defined in Eq. (3.3) with parameters for the SPS 200 MHz and 800 MHz cavities given in Tab. 3.1. The resonant frequency f_r , the shunt impedance $R_{\rm sh}$ and the quality factor Q are given together with the value of $R_{\rm sh}/Q$ and the e-folding time of the wakefield $2Q/\omega_r$ defined in Eq. (2.62).

A.1 HOMs of the SPS TW Structures

Table A.1 – Table of resonators for the total impedance of the HOMs of the 800 MHz TW structures. The total number of elements in the model is 2.

f_r [GHz]	$R_{\rm sh}$ [k Ω]	Q	$R_{\rm sh}/Q$ [Ω]	$2Q/\omega_r$ [ns]
1.878	44.0	850.0	51.8	144.0
1.926	44.0	2500.0	17.6	413.2
1.933	460.0	2500.0	184.0	411.7
1.935	300.0	2000.0	150.0	329.1
1.935	464.0	3000.0	154.7	493.5
1.936	60.0	2500.0	24.0	411.1
1.937	120.0	2500.0	48.0	410.8

f_r [GHz]	$R_{\rm sh} [{\rm k}\Omega]$	Q	$R_{ m sh}/Q$ [Ω]	$2Q/\omega_r$ [ns]
0.550	55.2	200.0	276.0	115.8
0.629	80.0	700.0	114.3	354.3
0.630	220.0	250.0	880.0	126.3
0.655	108.0	2500.0	43.2	1214.2
0.914	770.0	5000.0	154.0	1741.3
0.915	772.0	5000.0	154.4	1739.8
0.991	72.0	1500.0	48.0	481.9
1.131	110.0	4000.0	27.5	1126.2
1.132	132.0	4000.0	33.0	1124.9
1.133	156.0	5000.0	31.2	1404.6
1.133	160.0	5000.0	32.0	1404.1
1.188	96.0	7500.0	12.8	2009.9
1.209	106.0	6000.0	17.7	1579.3
1.450	130.0	5500.0	23.6	1207.4
1.507	174.0	8000.0	21.8	1689.4
1.508	236.0	8000.0	29.5	1689.1

Table A.2 – Table of resonators for the total impedance of the HOMs of the four-section 200 MHz TW structures. The total number of elements in the model is 2.

Table A.3 – Table of resonators for the total impedance of the five-section 200 MHz TW structures. The total number of elements in the model is 1.

f_r [GHz]	$R_{\rm sh}$ [k Ω]	Q	$R_{ m sh}/Q$ [Ω]	$2Q/\omega_r$ [ns]
0.550	66.0	150.0	440.0	86.8
0.629	260.0	300.0	866.7	151.8
0.630	140.0	500.0	280.0	252.7
0.656	136.0	2500.0	54.4	1213.8
0.914	940.0	4500.0	208.9	1566.8
0.915	720.0	4500.0	160.0	1565.8
0.991	96.0	2000.0	48.0	642.7
1.132	520.0	4000.0	130.0	1125.1
1.133	196.0	4500.0	43.6	1264.0
1.450	100.0	2000.0	50.0	439.1
1.507	240.0	7000.0	34.3	1478.2
1.508	184.0	7000.0	26.3	1477.8
1.538	132.0	6000.0	22.0	1241.6
1.539	160.0	6000.0	26.7	1241.1

A.2 QD-Type Flanges

Table A.4 – Table of resonators for the total impedance of the BPV-QD flanges. The total number of elements in the model is 90.

f_r [GHz]	$R_{\rm sh}$ [k Ω]	Q	$R_{\rm sh}/Q$ [Ω]	$2Q/\omega_r$ [ns]
1.060	56.7	896.8	63.2	269.4
1.083	63.5	775.0	82.0	227.8
1.302	0.1	1.2	110.7	0.3
1.881	664.1	774.0	858.0	131.0
2.122	1.9	8.0	241.8	1.2
2.179	22.5	1061.2	21.2	155.0
2.271	1547.9	1482.2	1044.4	207.7

Table A.5 – Table of resonators for the total impedance of the QD-QD flange. The total number of elements in the model is 75.

f_r [GHz]	$R_{\rm sh} [{\rm k}\Omega]$	Q	$R_{ m sh}/Q$ [Ω]	$2Q/\omega_r$ [ns]
1.760	1402.0	1050.0	1335.2	189.9
2.453	1290.0	1415.0	911.7	183.6

Table A.6 – Table of resonators for the total impedance of the QD-QD flange with enamel. The total number of elements in the model is 99.

f_r [GHz]	$R_{\rm sh}$ [k Ω]	Q	$R_{\rm sh}/Q$ [Ω]	$2Q/\omega_r$ [ns]
1.564	24.8	80.0	310.0	16.3
1.820	297.0	208.0	1427.9	36.4
2.456	1970.1	1380.0	1427.6	178.9

Table A.7 – Table of resonators for the total impedance of the QD-type shielded pumping ports. The total number of elements in the model is 71.

f_r [GHz]	$R_{\rm sh}$ [k Ω]	Q	$R_{\rm sh}/Q$ [Ω]	$2Q/\omega_r$ [ns]
2.302	330.0	480.0	687.5	66.4

A.3 QF-Type Flanges

Table A.8 – Table of resonators for the total impedance of the BPH-QF flanges. The total number of elements in the model is 12.

f_r [GHz]	$R_{\rm sh} [{\rm k}\Omega]$	Q	$R_{ m sh}/Q~[\Omega]$	$2Q/\omega_r$ [ns]
1.280	307.0	400.0	767.5	99.5
1.620	37.0	120.0	308.3	23.6

Table A.9 – Table of resonators for the total impedance of the BPH-QF flanges with damping resistor. The total number of elements in the model is 25.

<i>f_r</i> [GHz]	$R_{\rm sh}$ [k Ω]	Q	$R_{\rm sh}/Q$ [Ω]	$2Q/\omega_r$ [ns]
1.280	320.0	200.0	1600.0	49.7
1.620	39.0	60.0	650.0	11.8

Table A.10 – Table of resonators for the total impedance of the MBA-MBA flanges. The total number of elements in the model is 2.

f_r [GHz]	$R_{\rm sh} [{\rm k}\Omega]$	Q	$R_{\rm sh}/Q$ [Ω]	$2Q/\omega_r$ [ns]
1.415	42.6	270.0	157.8	60.7

Table A.11 – Table of resonators for the total impedance of the MBA-MBA flanges with damping resistor. The total number of elements in the model is 12.

f_r [GHz]	$R_{\rm sh}$ [k Ω]	Q	$R_{\rm sh}/Q~[\Omega]$	$2Q/\omega_r$ [ns]
1.415	58.5	75.0	780.0	16.9

Table A.12 – Table of resonators for the total impedance of the QF-MBA flanges without bellow. The total number of elements in the model is 2.

f_r [GHz]	$R_{\rm sh} [{\rm k}\Omega]$	Q	$R_{\rm sh}/Q~[\Omega]$	$2Q/\omega_r$ [ns]
1.593	54.0	930.0	58.1	185.8

Table A.13 – Table of resonators for the total impedance of the QF-MBA flanges with bellow and damping resistor. The total number of elements in the model is 1.

f_r [GHz]	$R_{\rm sh}$ [k Ω]	Q	$R_{ m sh}/Q$ [Ω]	$2Q/\omega_r$ [ns]
1.395	15.8	200.0	79.0	45.6
2.360	3.5	246.0	14.4	33.2

Table A.14 – Table of resonators for the total impedance of the QF-MBA flanges. The total number of elements in the model is 2.

f_r [GHz]	$R_{\rm sh}$ [k Ω]	Q	$R_{\rm sh}/Q$ [Ω]	$2Q/\omega_r$ [ns]
1.415	42.6	270.0	157.8	60.7

Table A.15 – Table of resonators for the total impedance of the QF-MBA flanges with damping resistor. The total number of elements in the model is 78.

f_r [GHz]	$R_{\rm sh} [{\rm k}\Omega]$	Q	$R_{ m sh}/Q$ [Ω]	$2Q/\omega_r$ [ns]
1.415	380.0	75.0	5066.6	16.9

Table A.16 – Table of resonators for the total impedance of the QF-MBA unshielded pumping ports. The total number of elements in the model is 25.

f_r [GHz]	$R_{\rm sh}$ [k Ω]	Q	$R_{\rm sh}/Q$ [Ω]	$2Q/\omega_r$ [ns]
1.408	17.0	80.0	212.8	18.1
1.499	131.6	65.0	2024.7	13.8
1.942	112.5	100.0	1125.0	16.4
1.985	6.2	130.0	48.1	20.8

Table A.17 – Table of resonators for the total impedance of the QF-QF flanges without bellow. The total number of elements in the model is 18.

f_r [GHz]	$R_{\rm sh}$ [k Ω]	Q	$R_{\rm sh}/Q~[\Omega]$	$2Q/\omega_r$ [ns]
1.593	486.0	930.0	522.6	185.8

Table A.18 – Table of resonators for the total impedance of the QF-QF flanges with bellow. The total number of elements in the model is 3.

f_r [GHz]	$R_{\rm sh}$ [k Ω]	Q	$R_{\rm sh}/Q$ [Ω]	$2Q/\omega_r$ [ns]
1.404	308.1	1300.0	237.0	294.7
2.360	69.1	1600.0	43.2	215.8

Table A.19 – Table of resonators for the total impedance of the QF-QF flanges with bellow and damping resistor. The total number of elements in the model is 22.

f_r [GHz]	$R_{\rm sh}$ [k Ω]	Q	$R_{ m sh}/Q$ [Ω]	$2Q/\omega_r$ [ns]
1.395	347.6	200.0	1738.0	45.6
2.360	78.0	246.0	316.9	33.2

A.4 Sector Valves and Beam Instrumentation

Table A.20 – Table of resonators for the total impedance of the VVSA sector valves. The total number of elements in the model is 28.

f_r [GHz]	$R_{\rm sh}$ [k Ω]	Q	$R_{\rm sh}/Q$ [Ω]	$2Q/\omega_r$ [ns]
0.608	34.4	360.0	95.5	188.5
1.270	224.0	600.0	373.3	150.4
1.324	83.2	492.0	169.1	118.3
1.427	19.9	528.0	37.8	117.8
1.797	13.2	331.0	40.0	58.6
2.394	17.4	1179.0	14.8	156.8

Table A.21 – Table of resonators for the total impedance of the VVSB-type sector valves. The total number of elements in the model is 36.

f_r [GHz]	$R_{\rm sh} [{\rm k}\Omega]$	Q	$R_{\rm sh}/Q$ [Ω]	$2Q/\omega_r$ [ns]
0.517	79.7	466.0	171.1	286.9
0.992	330.3	659.0	501.1	211.5
1.185	109.5	2686.0	40.8	721.5

f_r [GHz]	$R_{\rm sh}$ [k Ω]	Q	$R_{\rm sh}/Q$ [Ω]	$2Q/\omega_r$ [ns]
0.840	0.6	10.0	60.4	3.8
1.066	69.8	500.0	139.6	149.3
1.076	87.2	500.0	174.4	147.9
1.608	20.4	40.0	510.1	7.9
1.884	109.0	500.0	218.0	84.5
2.218	9.1	15.0	606.3	2.2

Table A.22 – Table of resonators for the total impedance of the beam position monitor horizontal (BPH). The total number of elements in the model is 106.

A.5 Kickers

Table A.23 – Table of resonators for the total impedance of the MBMKEL kickers. The total number of elements in the model is 4.

f_r [GHz]	$R_{\rm sh}$ [k Ω]	Q	$R_{\rm sh}/Q$ [Ω]	$2Q/\omega_r$ [ns]
0.052	4.0	5.8	686.5	35.3
0.056	4.0	3.5	1151.5	19.6
0.100	0.4	0.8	516.6	2.5
0.813	1.7	0.5	3386.4	0.2
0.876	1.5	1.8	819.2	0.6
1.222	3.0	1.3	2316.4	0.3
1.717	3.0	1.3	2260.8	0.2
2.439	2.4	1.1	2193.5	0.1

Table A.24 – Table of resonators for the total impedance of the MBMKES kickers. The total number of elements in the model is 3.

f_r [GHz]	$R_{\rm sh}$ [k Ω]	Q	$R_{ m sh}/Q$ [Ω]	$2Q/\omega_r$ [ns]
0.054	3.0	5.7	525.9	33.6
0.058	3.0	3.6	831.8	19.8
0.090	0.4	1.1	336.8	3.8
0.547	1.4	1.0	1415.6	0.6
0.870	1.0	1.6	648.4	0.6
1.217	3.0	1.1	2814.7	0.3
1.819	3.0	1.1	2775.4	0.2

Appendix A. Resonator Models of the Longitudinal SPS Impedance Before LIU Upgrades

Table A.25 – Table of resonators for the total impedance of the MKDH kicker at position 11751 and 11754. The total number of elements in the model is 2.

f_r [GHz]	$R_{\rm sh} [{\rm k}\Omega]$	Q	$R_{\rm sh}/Q$ [Ω]	$2Q/\omega_r$ [ns]
1.690	0.9	0.7	1276.0	0.1
1.029	3.2	1.0	3133.2	0.3

Table A.26 – Table of resonators for the total impedance of the MKDH kicker at position 11757. The total number of elements in the model is 1.

f_r [GHz]	$R_{\rm sh} [{\rm k}\Omega]$	Q	$R_{\rm sh}/Q$ [Ω]	$2Q/\omega_r$ [ns]
1.636	0.4	0.7	605.2	0.1
0.963	1.6	1.0	1556.5	0.3

Table A.27 – Table of resonators for the total impedance of the MKDV kicker at position 11731. The total number of elements in the model is 1.

f_r [GHz]	$R_{\rm sh}$ [k Ω]	Q	$R_{\rm sh}/Q$ [Ω]	$2Q/\omega_r$ [ns]
1.838	0.5	0.9	489.3	0.2

Table A.28 – Table of resonators for the total impedance of the MKDV kicker at position 11736. The total number of elements in the model is 1.

f_r [GHz]	$R_{\rm sh} [{\rm k}\Omega]$	Q	$R_{ m sh}/Q$ [Ω]	$2Q/\omega_r$ [ns]
1.849	0.3	0.9	313.3	0.2

Table A.29 – Table of resonators for the total impedance of the MKPA kickers at position 11931 and 11936. The total number of elements in the model is 2.

f_r [GHz]	$R_{\rm sh}$ [k Ω]	Q	$R_{\rm sh}/Q$ [Ω]	$2Q/\omega_r$ [ns]
1.892	1.7	0.5	3438.0	0.1
0.830	4.0	1.8	2155.9	0.7
1.009	3.3	1.1	2922.8	0.4
0.178	0.5	1.0	475.5	1.8
0.602	1.4	3.1	453.6	1.6
0.339	1.0	10.4	94.1	9.8
0.314	0.5	5.6	91.3	5.6

f_r [GHz]	$R_{\rm sh}$ [k Ω]	Q	$R_{\rm sh}/Q$ [Ω]	$2Q/\omega_r$ [ns]
1.882	0.4	0.5	864.0	0.1
0.607	0.3	3.5	93.5	1.8
1.011	0.8	1.1	719.3	0.4
0.184	0.1	0.9	131.8	1.6
0.828	1.0	1.8	554.4	0.7
0.333	0.3	6.9	44.9	6.6

Table A.30 – Table of resonators for the total impedance of the MKPC kicker at position 11952. The total number of elements in the model is 1.

Table A.31 – Table of resonators for the total impedance of the MKP kicker at position 11955. The total number of elements in the model is 1.

f_r [GHz]	$R_{\rm sh} [{\rm k}\Omega]$	Q	$R_{ m sh}/Q$ [Ω]	$2Q/\omega_r$ [ns]
0.660	3.6	1.6	2289.3	0.7
0.303	0.7	10.8	64.8	11.3
0.990	1.6	0.7	2393.9	0.2
0.164	0.3	1.1	322.5	2.1
0.511	2.2	3.0	717.7	1.9
0.436	1.1	3.0	353.8	2.2
0.279	0.6	4.6	130.7	5.2

Table A.32 – Table of resonators for the total impedance of the MKQH kicker at position 11653. The total number of elements in the model is 1.

f_r [GHz]	$R_{\rm sh}$ [k Ω]	Q	$R_{\rm sh}/Q$ [Ω]	$2Q/\omega_r$ [ns]
0.600	1.3	1.2	1060.5	0.6
2.019	0.8	0.5	1523.8	0.1
0.803	1.1	1.0	1045.8	0.4

A.6 Miscellaneous

Table A.33 – Table of resonators for the total impedance of the beam scrapers. The total number of elements in the model is 3.

f_r [GHz]	$R_{\rm sh}$ [k Ω]	Q	$R_{\rm sh}/Q$ [Ω]	$2Q/\omega_r$ [ns]
0.395	19.8	390.0	50.8	314.7
0.780	67.8	1080.0	62.8	440.8
0.965	73.2	2080.0	35.2	686.2
1.016	5.0	300.0	16.5	94.0
1.067	14.8	500.0	29.7	149.2
1.321	13.6	400.0	33.9	96.4
1.619	3.0	100.0	30.4	19.7
2.000	0.8	5.0	168.0	0.8

Table A.34 – Table of resonators for the total impedance of the tank of the beam scrapers. The total number of elements in the model is 2.

f_r [GHz]	$R_{\rm sh}$ [k Ω]	Q	$R_{\rm sh}/Q$ [Ω]	$2Q/\omega_r$ [ns]
0.917	108.4	2450.0	44.2	850.4
1.312	71.4	2380.0	30.0	577.4
2.500	0.1	1.0	140.0	0.1

Table A.35 – Table of resonators for the total impedance of the Y-chambers. The total number of elements in the model is 3.

f_r [GHz]	$R_{\rm sh}$ [k Ω]	Q	$R_{\rm sh}/Q$ [Ω]	$2Q/\omega_r$ [ns]
0.696	282.0	7400.0	38.1	3384.3
0.910	133.0	8415.0	15.8	2943.5
1.219	1.0	90.0	11.1	23.5
2.200	0.3	3.0	100.0	0.4

f_r [GHz]	$R_{\rm sh} [{\rm k}\Omega]$	Q	$R_{ m sh}/Q$ [Ω]	$2Q/\omega_r$ [ns]
0.095	48.4	164.5	294.4	549.7
0.105	20.9	182.2	114.9	554.8
0.115	12.8	201.3	63.3	558.2
0.197	10.5	336.1	31.2	543.1
0.212	31.2	318.8	97.9	478.7
0.532	10.4	673.6	15.4	403.4
0.544	14.1	830.4	17.0	486.0
0.549	32.9	722.2	45.6	418.8
0.911	12.8	875.8	14.6	306.1
0.945	17.0	1579.6	10.7	532.1
0.950	66.7	1304.3	51.1	437.0
0.952	36.7	703.1	52.2	235.1
0.954	48.5	942.0	51.5	314.3
0.956	27.7	863.9	32.1	287.6
1.132	10.6	833.5	12.7	234.3
1.135	22.9	1338.8	17.1	375.6
1.145	18.9	875.2	21.6	243.4
1.147	27.9	793.1	35.2	220.1
1.149	17.0	893.4	19.0	247.5
1.626	16.8	1159.1	14.5	226.9

Table A.36 – Table of resonators for the total impedance of the electromagnetic septa (ZS). The total number of elements in the model is 1.

B Resonator Models of the Longitudinal SPS Impedance After LIU Upgrades

In this appendix, the parameters of the resonators for the longitudinal SPS impedance model after LIU upgrades are given. Resonators with a value of $R_{\rm sh}/Q$ smaller than 10 Ω and a resonant frequency larger than 2.5 GHz are not shown. The fundamental impedance of the TW structures, which are not described by resonators, was defined in Eq. (3.3) with parameters for the SPS 200 MHz and 800 MHz cavities given in Tab. 3.1. The resonant frequency f_r , the shunt impedance $R_{\rm sh}$ and the quality factor Q are given together with the value of $R_{\rm sh}/Q$ and the e-folding time of the wakefield $2Q/\omega_r$ defined in Eq. (2.62).

B.1 HOMs of the SPS TW Structures

Table B.1 – Table of resonators for the total impedance of the HOMs of the 800 MHz TW structures. The total number of elements in the model is 2.

f_r [GHz]	$R_{\rm sh} [{\rm k}\Omega]$	Q	$R_{\rm sh}/Q$ [Ω]	$2Q/\omega_r$ [ns]
1.878	44.0	850.0	51.8	144.0
1.926	44.0	2500.0	17.6	413.2
1.933	460.0	2500.0	184.0	411.7
1.935	300.0	2000.0	150.0	329.1
1.935	464.0	3000.0	154.7	493.5
1.936	60.0	2500.0	24.0	411.1
1.937	120.0	2500.0	48.0	410.8

f_r [GHz]	$R_{\rm sh} [{\rm k}\Omega]$	Q	$R_{\rm sh}/Q$ [Ω]	$2Q/\omega_r$ [ns]
0.550	66.2	120.0	552.0	69.5
0.629	96.0	420.0	228.6	212.6
0.630	264.0	150.0	1760.0	75.8
0.655	129.6	1500.0	86.4	728.5
0.914	924.0	3000.0	308.0	1044.8
0.915	926.4	3000.0	308.8	1043.9
0.991	86.4	900.0	96.0	289.2
1.131	132.0	2400.0	55.0	675.7
1.132	158.4	2400.0	66.0	674.9
1.133	187.2	3000.0	62.4	842.8
1.133	192.0	3000.0	64.0	842.5
1.188	115.2	4500.0	25.6	1205.9
1.209	127.2	3600.0	35.3	947.6
1.450	156.0	3300.0	47.3	724.4
1.507	208.8	4800.0	43.5	1013.7
1.508	283.2	4800.0	59.0	1013.5

Table B.2 – Table of resonators for the total impedance of HOMs of the three-section 200 MHz TW structures. The total number of elements in the model is 4.

Table B.3 – Table of resonators for the total impedance of the HOMs of the four-section 200 MHz TW structures. The total number of elements in the model is 2.

f_r [GHz]	$R_{\rm sh}$ [k Ω]	Q	$R_{\rm sh}/Q$ [Ω]	$2Q/\omega_r$ [ns]
0.550	55.2	200.0	276.0	115.8
0.629	80.0	700.0	114.3	354.3
0.630	220.0	250.0	880.0	126.3
0.655	108.0	2500.0	43.2	1214.2
0.914	770.0	5000.0	154.0	1741.3
0.915	772.0	5000.0	154.4	1739.8
0.991	72.0	1500.0	48.0	481.9
1.131	110.0	4000.0	27.5	1126.2
1.132	132.0	4000.0	33.0	1124.9
1.133	156.0	5000.0	31.2	1404.6
1.133	160.0	5000.0	32.0	1404.1
1.188	96.0	7500.0	12.8	2009.9
1.209	106.0	6000.0	17.7	1579.3
1.450	130.0	5500.0	23.6	1207.4
1.507	174.0	8000.0	21.8	1689.4
1.508	236.0	8000.0	29.5	1689.1
B.2 QD-Type Flanges

Table B.4 – Table of resonators for the total impedance of the BPV-QD flanges. The total number of elements in the model is 90.

f_r [GHz]	$R_{\rm sh}$ [k Ω]	Q	$R_{\rm sh}/Q$ [Ω]	$2Q/\omega_r$ [ns]
1.060	56.7	896.8	63.2	269.4
1.083	63.5	775.0	82.0	227.8
1.302	0.1	1.2	110.7	0.3
1.881	664.1	774.0	858.0	131.0
2.122	1.9	8.0	241.8	1.2
2.179	22.5	1061.2	21.2	155.0
2.271	1547.9	1482.2	1044.4	207.7

Table B.5 – Table of resonators for the total impedance of the QD-QD flanges. The total number of elements in the model is 75.

f_r [GHz]	$R_{\rm sh} [{\rm k}\Omega]$	Q	$R_{ m sh}/Q$ [Ω]	$2Q/\omega_r$ [ns]
1.760	1402.0	1050.0	1335.2	189.9
2.453	1290.0	1415.0	911.7	183.6

Table B.6 – Table of resonators for the total impedance of the QD-QD flanges with enamel. The total number of elements in the model is 99.

f_r [GHz]	$R_{\rm sh}$ [k Ω]	Q	$R_{\rm sh}/Q$ [Ω]	$2Q/\omega_r$ [ns]
1.564	24.8	80.0	310.0	16.3
1.820	297.0	208.0	1427.9	36.4
2.456	1970.1	1380.0	1427.6	178.9

Table B.7 – Table of resonators for the total impedance of the QD-type shielded pumping ports. The total number of elements in the model is 71.

f_r [GHz]	$R_{\rm sh}$ [k Ω]	Q	$R_{\rm sh}/Q$ [Ω]	$2Q/\omega_r$ [ns]
2.302	330.0	480.0	687.5	66.4

B.3 QF-Type Flanges

Table B.8 – Table of resonators for the total impedance of the QF-MBA flanges with shield. The total number of elements in the model is 94.

f_r [GHz]	$R_{\rm sh}$ [k Ω]	Q	$R_{\rm sh}/Q$ [Ω]	$2Q/\omega_r$ [ns]
1.152	32.1	80.0	401.3	22.1
1.987	2.2	100.0	22.0	16.0

Table B.9 – Table of resonators for the total impedance of the QF-MBA unshielded pumping ports. The total number of elements in the model is 10.

f_r [GHz]	$R_{\rm sh}$ [k Ω]	Q	$R_{\rm sh}/Q~[\Omega]$	$2Q/\omega_r$ [ns]
1.408	6.8	80.0	85.1	18.1
1.499	52.6	65.0	809.9	13.8
1.942	45.0	100.0	450.0	16.4
1.985	2.5	130.0	19.2	20.8

Table B.10 – Table of resonators for the total impedance of the QF-QF flanges with shield. The total number of elements in the model is 25.

f_r [GHz]	$R_{\rm sh}$ [k Ω]	Q	$R_{\rm sh}/Q$ [Ω]	$2Q/\omega_r$ [ns]
1.152	15.4	145.0	106.1	40.1

Table B.11 – Table of resonators for the total impedance of the QF-QF flange without bellow with shield. The total number of elements in the model is 18.

f_r [GHz]	$R_{\rm sh}$ [k Ω]	Q	$R_{\rm sh}/Q$ [Ω]	$2Q/\omega_r$ [ns]
1.511	17.4	210.0	82.7	44.2

f_r [GHz]	$R_{\rm sh}$ [k Ω]	Q	$R_{ m sh}/Q$ [Ω]	$2Q/\omega_r$ [ns]
0.840	0.6	10.0	60.4	3.8
1.066	69.8	500.0	139.6	149.3
1.076	87.2	500.0	174.4	147.9
1.608	20.4	40.0	510.1	7.9
1.884	109.0	500.0	218.0	84.5
2.218	9.1	15.0	606.3	2.2

B.4 Sector Valves and Beam Instrumentation

Table B.12 – Table of resonators for the total impedance of the beam position horizontal (BPH). The total number of elements in the model is 106.

Table B.13 – Table of resonators for the total impedance of the VVSA-type sector valves. The total number of elements in the model is 28.

f_r [GHz]	$R_{\rm sh} [{\rm k}\Omega]$	Q	$R_{\rm sh}/Q$ [Ω]	$2Q/\omega_r$ [ns]
0.608	34.4	360.0	95.5	188.5
1.270	224.0	600.0	373.3	150.4
1.324	83.2	492.0	169.1	118.3
1.427	19.9	528.0	37.8	117.8
1.797	13.2	331.0	40.0	58.6
2.394	17.4	1179.0	14.8	156.8

Table B.14 – Table of resonators for the total impedance of the VVSB-type sector valves. The total number of elements in the model is 36.

f_r [GHz]	$R_{\rm sh}$ [k Ω]	Q	$R_{\rm sh}/Q$ [Ω]	$2Q/\omega_r$ [ns]
0.517	79.7	466.0	171.1	286.9
0.992	330.3	659.0	501.1	211.5
1.185	109.5	2686.0	40.8	721.5

B.5 Kickers

Table B.15 – Table of resonators for the total impedance of the MBMKEL kickers. The total number of elements in the model is 4.

f_r [GHz]	$R_{\rm sh}$ [k Ω]	Q	$R_{\rm sh}/Q$ [Ω]	$2Q/\omega_r$ [ns]
0.052	4.0	5.8	686.5	35.3
0.056	4.0	3.5	1151.5	19.6
0.100	0.4	0.8	516.6	2.5
0.813	1.7	0.5	3386.4	0.2
0.876	1.5	1.8	819.2	0.6
1.222	3.0	1.3	2316.4	0.3
1.717	3.0	1.3	2260.8	0.2
2.439	2.4	1.1	2193.5	0.1

Table B.16 – Table of resonators for the total impedance of the MBMKES kickers. The total number of elements in the model is 3.

f_r [GHz]	$R_{\rm sh}$ [k Ω]	Q	$R_{ m sh}/Q$ [Ω]	$2Q/\omega_r$ [ns]
0.054	3.0	5.7	525.9	33.6
0.058	3.0	3.6	831.8	19.8
0.090	0.4	1.1	336.8	3.8
0.547	1.4	1.0	1415.6	0.6
0.870	1.0	1.6	648.4	0.6
1.217	3.0	1.1	2814.7	0.3
1.819	3.0	1.1	2775.4	0.2

Table B.17 – Table of resonators for the total impedance of the MKDH kickers at position 11751 and 11754. The total number of elements in the model is 2.

f_r [GHz]	$R_{\rm sh}$ [k Ω]	Q	$R_{\rm sh}/Q$ [Ω]	$2Q/\omega_r$ [ns]
1.690	0.9	0.7	1276.0	0.1
1.029	3.2	1.0	3133.2	0.3

Table B.18 – Table of resonators for the total impedance of the MKDH kicker at position 11757. The total number of elements in the model is 1.

f_r [GHz]	$R_{\rm sh}$ [k Ω]	Q	$R_{\rm sh}/Q$ [Ω]	$2Q/\omega_r$ [ns]
1.636	0.4	0.7	605.2	0.1
0.963	1.6	1.0	1556.5	0.3

Table B.19 – Table of resonators for the total impedance of the MKDV kicker at position 11731. The total number of elements in the model is 1.

f_r [GHz]	$R_{\rm sh}$ [k Ω]	Q	$R_{\rm sh}/Q$ [Ω]	$2Q/\omega_r$ [ns]
1.838	0.5	0.9	489.3	0.2

Table B.20 – Table of resonators for the total impedance of the MKDV kicker at position 11736. The total number of elements in the model is 1.

f_r [GHz]	$R_{\rm sh} [{\rm k}\Omega]$	Q	$R_{\rm sh}/Q$ [Ω]	$2Q/\omega_r$ [ns]
1.849	0.3	0.9	313.3	0.2

Table B.21 – Table of resonators for the total impedance of the MKPA kickers at position 11931 and 11936. The total number of elements in the model is 2.

f_r [GHz]	$R_{\rm sh} [{ m k}\Omega]$	Q	$R_{\rm sh}/Q$ [Ω]	$2Q/\omega_r$ [ns]
1.892	1.7	0.5	3438.0	0.1
0.830	4.0	1.8	2155.9	0.7
1.009	3.3	1.1	2922.8	0.4
0.178	0.5	1.0	475.5	1.8
0.602	1.4	3.1	453.6	1.6
0.339	1.0	10.4	94.1	9.8
0.314	0.5	5.6	91.3	5.6

Table B.22 – Table of resonators for the total impedance of the MKPC kicker at position 11952. The total number of elements in the model is 1.

f_r [GHz]	$R_{\rm sh} [{\rm k}\Omega]$	Q	$R_{\rm sh}/Q$ [Ω]	$2Q/\omega_r$ [ns]
1.882	0.4	0.5	864.0	0.1
0.607	0.3	3.5	93.5	1.8
1.011	0.8	1.1	719.3	0.4
0.184	0.1	0.9	131.8	1.6
0.828	1.0	1.8	554.4	0.7
0.333	0.3	6.9	44.9	6.6

f_r [GHz]	$R_{\rm sh}$ [k Ω]	Q	$R_{\rm sh}/Q$ [Ω]	$2Q/\omega_r$ [ns]
0.660	3.6	1.6	2289.3	0.7
0.303	0.7	10.8	64.8	11.3
0.990	1.6	0.7	2393.9	0.2
0.164	0.3	1.1	322.5	2.1
0.511	2.2	3.0	717.7	1.9
0.436	1.1	3.0	353.8	2.2
0.279	0.6	4.6	130.7	5.2

Table B.23 – Table of resonators for the total impedance of the MKP kicker at position 11955. The total number of elements in the model is 1.

Table B.24 – Table of resonators for the total impedance of the MKQH kicker at position 11653. The total number of elements in the model is 1.

f_r [GHz]	$R_{\rm sh} [{\rm k}\Omega]$	Q	$R_{\rm sh}/Q$ [Ω]	$2Q/\omega_r$ [ns]
0.600	1.3	1.2	1060.5	0.6
2.019	0.8	0.5	1523.8	0.1
0.803	1.1	1.0	1045.8	0.4

B.6 Miscellaneous

Table B.25 – Table of resonators for the total impedance of the beam scrapers. The total number of elements in the model is 3.

f_r [GHz]	$R_{\rm sh}$ [k Ω]	Q	$R_{\rm sh}/Q$ [Ω]	$2Q/\omega_r$ [ns]
0.395	19.8	390.0	50.8	314.7
0.780	67.8	1080.0	62.8	440.8
0.965	73.2	2080.0	35.2	686.2
1.016	5.0	300.0	16.5	94.0
1.067	14.8	500.0	29.7	149.2
1.321	13.6	400.0	33.9	96.4
1.619	3.0	100.0	30.4	19.7
2.000	0.8	5.0	168.0	0.8

f_r [GHz]	$R_{\rm sh}$ [k Ω]	Q	$R_{ m sh}/Q$ [Ω]	$2Q/\omega_r$ [ns]
0.917	108.4	2450.0	44.2	850.4
1.312	71.4	2380.0	30.0	577.4
2.500	0.1	1.0	140.0	0.1

Table B.26 – Table of resonators for the total impedance of the tank of the beam scrapers. The total number of elements in the model is 2.

Table B.27 – Table of resonators for the total impedance of the Y-chambers. The total number of elements in the model is 3.

f_r [GHz]	$R_{\rm sh}$ [k Ω]	Q	$R_{\rm sh}/Q$ [Ω]	$2Q/\omega_r$ [ns]
0.696	282.0	7400.0	38.1	3384.3
0.910	133.0	8415.0	15.8	2943.5
1.219	1.0	90.0	11.1	23.5
2.200	0.3	3.0	100.0	0.4

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Personal Skills

Flexibility Communication Enthusiasm Hard-working Passionate

Accelerator Physics

RF systems

Beam optics

Impedance

Synchrotron operations

Languages

English ****

German ★★★★

Data acquisition

Joël Repond

Accelerator Physicist

Experience

since 2016 **PhD student** European Organization for Nuclear Research, Switzerland Participation to many beam studies related to the study of the longitudinal intensity limitations in the Super Proton Synchrotron at CERN. Publications and presentations in international conferences.

since 2016

5 **Scientific secretary** European Organization for Nuclear Research, Switzerland Website management and redaction of the minutes for the LIU-SPS Beam Dynamics Working Group meetings.

Internship Numerical study of the reheating stage of the early universe in the framework of the Higgs-Inflation scenario.

2013-2014 **Teaching assistant** Swiss Federal Institute of Technology in Lausanne, Switzerland Supervision of the exercise sessions in physics and numerical physics lectures for students of first and second year.

Education

- since 2016Ph.D. candidateEuropean Organization for Nuclear Research, Switzerland
Research in longitudinal beam dynamics applied to the intensity limitations of
the CERN SPS. Title of the thesis: Possible mitigations of longitudinal inten-
sity limitations for HL-LHC beam in the CERN SPS20175th CERN Thematic School of ComputingSplit,Croatia
 - Efficient Parallel Processing of Future Scientific Data
- 2012 2015 **Master of Science MSc** Swiss Federal Institute of Technology in Lausanne, Switzerland Main subjects: Particle Physics and Cosmology. Title of the thesis: *Preheating*. Minor in Computational Science and Engineering.

Publications

J. Repond, H. Damerau, A. Lasheen, E. Shaposhnikova, Lower-Harmonic RF System in the CERN SPS, in Proc. IPAC'18, pages 3087–3090, Vancouver, BC Canada, 29 Apr - 4 Mar 2018.

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